VTB

CORRECCIÓ D'ERRORS

99 f) $F = \begin{pmatrix} t+3 & 4 & 0 \\ 0 & t-1 & 1 \\ -4 & -4 & t-1 \end{pmatrix}$

$$|F| = (t+3)(t-1)^2 - 16 + 4(t+3)$$

$$= (t+3)(t^2-2t+1) - 16 + 4t + 12$$

$$= t^3 - 2t^2 + t + 3t^2 - 6t + 3 - 16 + 4t + 12$$

$$= t^3 + t^2 - t - 1 = 0$$

$$\text{Div}(1) = \{\pm 1\}$$

	1	1	-1	-1
1		1	2	1
	1	2	1	0

$\downarrow$

$t^2 + 2t + 1.$

$$t^3 + t^2 - t - 1$$

$$= (t-1)(t^2+2t+1)$$

$$t^2 + 2t + 1 = 0$$

$$t = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm 0}{2} \left< \begin{matrix} -1 \\ -1 \end{matrix} \right.$$

$$\Rightarrow t^3 + t^2 - t - 1 = (t-1)(t+1)^2$$

$\Rightarrow$ arrels 1 i -1.

- Si $t \neq 1$ i $t \neq -1 \Rightarrow |F| \neq 0 \Rightarrow rg\, F = 3$
- Si $t = 1 \Rightarrow |F| = 0 \Rightarrow F = \begin{pmatrix} 4 & 4 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & 0 \end{pmatrix}$

$\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = 4 \neq 0 \Rightarrow rg\, F = 2$

• Si $t=-1 \Rightarrow |F|=0 \Rightarrow \nexists F^{-1}$

$$F=\begin{pmatrix} 2 & 4 & 0 \\ 0 & -2 & 1 \\ -4 & -4 & -2 \end{pmatrix} \qquad \begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = -4 \neq 0$$

$$\Downarrow$$

$$\exists F=2$$

g) $$G=\begin{pmatrix} t & 1 & 1 & 2 \\ 2 & t & t^2 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

Sabemos que $\exists G \leq 3$.

Sean

$$\Delta = \begin{vmatrix} t & 1 & 2 \\ 2 & t & 1 \\ 2 & 1 & 2 \end{vmatrix} = 2t^2+2+4-4t-4-t$$

$$= 2t^2-5t+2$$

$$2t^2-5t+2=0$$

$$t=\frac{5\pm\sqrt{25-4\cdot 2\cdot 2}}{2\cdot 2}=\frac{5\pm 3}{4} \begin{cases} 2 \\ \frac{2}{4}=\frac{1}{2} \end{cases}$$

• Si $t\neq 2$; $t\neq \frac{1}{2} \Rightarrow |\Delta| \neq 0 \Rightarrow \exists G = 3$

• Si $t=2 \Rightarrow \Delta=0$; $G=\begin{pmatrix} 2 & 1 & 1 & 2 \\ 2 & 2 & 4 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$

Sean

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 2 & 2 & 4 \\ 2 & 1 & 1 \end{vmatrix} = 4+8+2-4-2-8=0$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 2 \\ 2 & 4 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 16+2+4-16-4-2=0$$

$$\Delta_4 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 8+1+4-8-4-1=0$$

$\Rightarrow$ Tots els menors d'ordre 3 són nuls $\Rightarrow$ vs $6 = 2$, ja

que $\begin{vmatrix} 2 & 1 \\ 2 & 2 \end{vmatrix} = 4 - 2 = 2 \neq 0$