

The Open Source CFD Toolbox

# **Programmer's Guide**

Version 2.0.0 16th June 2011 Copyright © 2004-2011 OpenCFD Limited.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 published by the Free Software Foundation; with no Invariant Sections, no Back-Cover Texts and one Front-Cover Text: "Available free from openfoam.org." A copy of the license is included in the section entitled "GNU Free Documentation License".

This document is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

### **GNU Free Documentation License**

Version 1.2, November 2002 Copyright ©2000,2001,2002 Free Software Foundation, Inc.

59 Temple Place, Suite 330, Boston, MA 02111-1307 USA

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

#### Preamble

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondarily, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

#### 1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The **"Document"**, below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as **"you"**. You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "Modified Version" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "Secondary Section" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The "Invariant Sections" are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The "Cover Texts" are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A "**Transparent**" copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called "**Opaque**".

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machinegenerated HTML, PostScript or PDF produced by some word processors for output purposes only.

The "**Title Page**" means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

A section "Entitled XYZ" means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as "Acknowledgements", "Dedications", "Endorsements", or "History".) To "Preserve the Title" of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

#### 2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

#### 3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

#### 4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

- A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
- B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
- C. State on the Title page the name of the publisher of the Modified Version, as the publisher.
- D. Preserve all the copyright notices of the Document.
- E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.
- F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
- G. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
- H. Include an unaltered copy of this License.
- I. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.

- J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.
- K. For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
- L. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
- M. Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.
- N. Do not retitle any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.
- O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

#### 5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements".

#### 6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

#### 7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

#### 8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

#### 9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sublicense or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

#### **10. FUTURE REVISIONS OF THIS LICENSE**

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See http://www.gnu.org/copyleft/.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of

any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

### Trademarks

ANSYS is a registered trademark of ANSYS Inc. CFX is a registered trademark of Ansys Inc. CHEMKIN is a registered trademark of Reaction Design Corporation EnSight is a registered trademark of Computational Engineering International Ltd. Fieldview is a registered trademark of Intelligent Light Fluent is a registered trademark of Ansys Inc. GAMBIT is a registered trademark of Ansys Inc. Icem-CFD is a registered trademark of Ansys Inc. I-DEAS is a registered trademark of Structural Dynamics Research Corporation JAVA is a registered trademark of Sun Microsystems Inc. Linux is a registered trademark of Linus Torvalds OpenFOAM is a registered trademark of Kitware STAR-CD is a registered trademark of Computational Dynamics Ltd. UNIX is a registered trademark of The Open Group

# Contents

Co	opyri	ght No	otice	P-2
G	NU I	Free Do	ocumentation Licence	P-3
	1. A	PPLIC	ABILITY AND DEFINITIONS	P-3
	2. V	ERBAT	TIM COPYING	P-4
	3. C	OPYIN	G IN QUANTITY	P-4
	4. N	IODIFI	CATIONS	P-5
	5. C	OMBIN	NING DOCUMENTS	P-6
	6. C	OLLEC	TIONS OF DOCUMENTS	P-7
	7. A	GGRE	GATION WITH INDEPENDENT WORKS	P-7
	8. T	RANSI	ATION	P-7
	9. T	ERMIN	IATION	P-7
	10. ]	FUTUR	E REVISIONS OF THIS LICENSE	P-7
Tr	aden	narks		P-9
Co	onter	ıts		P-11
1	Ten	sor ma	thematics	P-15
	1.1	Coordi	nate system	P-15
	1.2	Tensor	<b>S</b>	P-15
		1.2.1	Tensor notation	P-17
	1.3	Algebr	aic tensor operations	P-17
		1.3.1	The inner product	P-18
		1.3.2	The double inner product of two tensors	P-19
		1.3.3	The triple inner product of two third rank tensors	P-19
		1.3.4	The outer product	P-19
		1.3.5	The cross product of two vectors	P-19
		1.3.6	Other general tensor operations	P-20
		1.3.7	Geometric transformation and the identity tensor	P-20
		1.3.8	Useful tensor identities	P-21
		1.3.9	Operations exclusive to tensors of rank 2	P-21
		1.3.10	Operations exclusive to scalars	P-22
	1.4	OpenF	OAM tensor classes	P-23
		1.4.1	Algebraic tensor operations in OpenFOAM	P-23
	1.5	Dimen	sional units	P-25
<b>2</b>	Disc	cretisat	tion procedures	P-27
	2.1	Differe	ntial operators	P-27
		2.1.1	Gradient	P-27
		2.1.2	Divergence	P-28

		2.1.3	Curl	P-28
		2.1.4	Laplacian	P-28
		2.1.5	Temporal derivative	P-28
	2.2	Overvi	iew of discretisation	P-29
		2.2.1	OpenFOAM lists and fields	P-29
	2.3	Discret	tisation of the solution domain	P-29
		2.3.1	Defining a mesh in OpenFOAM	P-31
		2.3.2	Defining a geometricField in OpenFOAM	P-32
	2.4	Equati	ion discretisation	P-33
		2.4.1	The Laplacian term	P-38
		2.4.2	The convection term	P-38
		2.4.3	First time derivative	P-39
		2.4.4	Second time derivative	P-39
		2.4.5	Divergence	P-39
		2.4.6	Gradient	P-40
		2.4.7	Grad-grad squared	P-41
		2.4.8	Curl	P-41
		2.4.9	Source terms	P-41
		2.4.10	Other explicit discretisation schemes	P-41
	2.5		oral discretisation	P-42
		2.5.1	Treatment of temporal discretisation in OpenFOAM	P-43
	2.6	Bound	ary Conditions	P-43
		2.6.1	Physical boundary conditions	P-44
3	Eva	mnles	of the use of OpenFOAM	P-45
Č	3.1	-	round a cylinder	P-45
	0.1	3.1.1	Problem specification	
				P-46
			-	P-46 P-47
		3.1.2	Note on potentialFoam	P-47
		$3.1.2 \\ 3.1.3$	Note on potentialFoam          Mesh generation	P-47 P-47
		$3.1.2 \\ 3.1.3 \\ 3.1.4$	Note on potentialFoam	P-47 P-47 P-49
	3.2	3.1.2 3.1.3 3.1.4 3.1.5	Note on potentialFoam	P-47 P-47 P-49 P-50
	3.2	3.1.2 3.1.3 3.1.4 3.1.5 Steady	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51
	3.2	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53
	3.2	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53 P-54
	3.2	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57
	3.2	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57 P-57
		3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57 P-57 P-58
	3.2 3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers	Note on potentialFoam	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57 P-57 P-58 P-58
		3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caser turbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specification	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57 P-57 P-58 P-58 P-58 P-58
		3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseturbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generation	P-47 P-47 P-49 P-50 P-51 P-53 P-54 P-57 P-57 P-57 P-58 P-58 P-58 P-58 P-58
		3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3	Note on potentialFoam	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseRunning the caseturbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingBoundary conditionsRunning the case and post-processingRunning the case and post-processingProblem specificationRunning the caseExercise	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \end{array}$
		3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decom	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseRunning the casev turbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingProblem specificationMesh generationProblem specificationMesh generationMesh generationProblem specificationMesh generationMesh generationMesh generationRunning the caseExercisePression of a tank internally pressurised with water	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \\ P-62 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decom 3.4.1	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseRunning the caseV turbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generationBunning the case and post-processingRunning the case and post-processingProblem specificationMesh generationProblem specificationProblem specification	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-62 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decom 3.4.1 3.4.2	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseturbulent flow over a backward-facing stepv turbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingProblem specificationMesh generationRunning the case and post-processingProblem specificationMesh generationRunning the caseExerciseProblem specificationMesh generationRunning the caseExerciseProblem specificationMesh GenerationMesh Generation	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-62 \\ P-63 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decom 3.4.1 3.4.2 3.4.3	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseturbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingProblem specificationBoundary conditions and initial fieldsCase controlRunning the case and post-processingProblem specificationMesh generationRunning the case and post-processingProblem specificationMesh generationRunning the caseProblem specificationProblem specificationMesh generationRunning the caseExerciseProblem specificationMesh GenerationProblem specificationProblem specificationProble	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-63 \\ P-63 \\ P-65 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decomp 3.4.1 3.4.2 3.4.3 3.4.4	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the casev turbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingSonic flow over a forward-facing stepProblem specificationMesh generationRunning the caseExerciseProblem specificationMesh generationRunning the caseExerciseProblem specificationProblem specificationPr	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-62 \\ P-63 \\ P-65 \\ P-66 \end{array}$
	3.3 3.4	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decomposition 3.4.1 3.4.2 3.4.3 3.4.4 3.4.5	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseturbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the caseProblem specificationMesh generationRunning the caseExerciseProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationMesh GenerationPreparing the RunRunning the caseImproving the solution by refining the mesh	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-63 \\ P-65 \\ P-66 \\ P-66 \\ P-66 \end{array}$
	3.3	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decom 3.4.1 3.4.2 3.4.3 3.4.4 3.4.5 Magne	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caserurbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the caseExerciseProblem specificationMesh generationRunning the caseExerciseProblem specificationProblem specificationProblem specificationRunning the caseImproving the solution by refining the meshMunning the caseImproving the solution by refining the mesh	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-54 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-62 \\ P-63 \\ P-65 \\ P-66 \\ P-66 \\ P-67 \end{array}$
	3.3 3.4	3.1.2 3.1.3 3.1.4 3.1.5 Steady 3.2.1 3.2.2 3.2.3 3.2.4 3.2.5 Supers 3.3.1 3.3.2 3.3.3 3.3.4 Decomposition 3.4.1 3.4.2 3.4.3 3.4.4 3.4.5	Note on potentialFoamMesh generationBoundary conditions and initial fieldsRunning the caseturbulent flow over a backward-facing stepProblem specificationMesh generationBoundary conditions and initial fieldsCase controlRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the case and post-processingsonic flow over a forward-facing stepProblem specificationMesh generationRunning the caseProblem specificationMesh generationRunning the caseExerciseProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationProblem specificationMesh GenerationPreparing the RunRunning the caseImproving the solution by refining the mesh	$\begin{array}{c} P-47 \\ P-47 \\ P-49 \\ P-50 \\ P-51 \\ P-53 \\ P-53 \\ P-57 \\ P-57 \\ P-57 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-58 \\ P-60 \\ P-61 \\ P-61 \\ P-61 \\ P-62 \\ P-62 \\ P-63 \\ P-65 \\ P-66 \\ P-66 \\ P-66 \end{array}$

	3.5.3	Running the case	P-70
Index			P-73

P-13

# Chapter 1

# **Tensor mathematics**

This Chapter describes tensors and their algebraic operations and how they are represented in mathematical text in this book. It then explains how tensors and tensor algebra are programmed in OpenFOAM.

## 1.1 Coordinate system

OpenFOAM is primarily designed to solve problems in continuum mechanics, *i.e.* the branch of mechanics concerned with the stresses in solids, liquids and gases and the deformation or flow of these materials. OpenFOAM is therefore based in 3 dimensional space and time and deals with physical entities described by tensors. The coordinate system used by OpenFOAM is the right-handed rectangular Cartesian axes as shown in Figure 1.1. This system of axes is constructed by defining an origin O from which three lines are drawn at right angles to each other, termed the Ox, Oy, Oz axes. A right-handed set of axes is defined such that to an observer looking down the Oz axis (with O nearest them), the arc from a point on the Ox axis to a point on the Oy axis is in a clockwise sense.

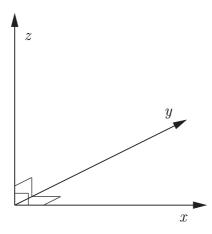


Figure 1.1: Right handed axes

### 1.2 Tensors

The term tensor describes an entity that belongs to a particular space and obeys certain mathematical rules. Briefly, tensors are represented by a set of *component values* relating to a set of unit base vectors; in OpenFOAM the unit base vectors  $\mathbf{i}_x$ ,  $\mathbf{i}_y$  and  $\mathbf{i}_z$  are

aligned with the right-handed rectangular Cartesian axes x, y and z respectively. The base vectors are therefore orthogonal, *i.e.* at right-angles to one another. Every tensor has the following attributes:

**Dimension** d of the particular space to which they belong, *i.e.* d = 3 in OpenFOAM;

**Rank** An integer  $r \ge 0$ , such that the number of component values  $= d^r$ .

While OpenFOAM 1.x is set to 3 dimensions, it offers tensors of ranks 0 to 3 as standard while being written in such a way to allow this basic set of ranks to be extended indefinitely. Tensors of rank 0 and 1, better known as scalars and vectors, should be familiar to readers; tensors of rank 2 and 3 may not be so familiar. For completeness all ranks of tensor offered as standard in OpenFOAM 1.x are reviewed below.

- **Rank 0 'scalar'** Any property which can be represented by a single real number, denoted by characters in italics, *e.g.* mass m, volume V, pressure p and viscosity  $\mu$ .
- **Rank 1 'vector'** An entity which can be represented physically by both magnitude and direction. In component form, the vector  $\mathbf{a} = (a_1, a_2, a_3)$  relates to a set of Cartesian axes x, y, z respectively. The *index notation* presents the same vector as  $a_i$ , i = 1, 2, 3, although the list of indices i = 1, 2, 3 will be omitted in this book, as it is intuitive since we are always dealing with 3 dimensions.
- **Rank 2 'tensor'** or second rank tensor, **T** has 9 components which can be expressed in array notation as:

$$\mathbf{T} = T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$$
(1.1)

The components  $T_{ij}$  are now represented using 2 indices since r = 2 and the list of indices i, j = 1, 2, 3 is omitted as before. The components for which i = j are referred to as the diagonal components, and those for which  $i \neq j$  are referred to as the off-diagonal components. The *transpose* of **T** is produced by exchanging components across the diagonal such that

$$\mathbf{T}^{\mathrm{T}} = T_{ji} = \begin{pmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{pmatrix}$$
(1.2)

Note: a rank 2 tensor is often colloquially termed 'tensor' since the occurrence of higher order tensors is fairly rare.

- Symmetric rank 2 The term 'symmetric' refers to components being symmetric about the diagonal, *i.e.*  $T_{ij} = T_{ji}$ . In this case, there are only 6 independent components since  $T_{12} = T_{21}$ ,  $T_{13} = T_{31}$  and  $T_{23} = T_{32}$ . OpenFOAM distinguishes between symmetric and non-symmetric tensors to save memory by storing 6 components rather than 9 if the tensor is symmetric. Most tensors encountered in continuum mechanics are symmetric.
- **Rank 3** has 27 components and is represented in index notation as  $P_{ijk}$  which is too long to represent in array notation as in Equation 1.1.
- **Symmetric rank 3** Symmetry of a rank 3 tensor is defined in OpenFOAM to mean that  $P_{ijk} = P_{ikj} = P_{jik} = P_{jki} = P_{kij} = P_{kji}$  and therefore has 10 independent components. More specifically, it is formed by the outer product of 3 identical vectors, where the outer product operation is described in Section 1.3.4.

#### 1.2.1 Tensor notation

This is a book on computational continuum mechanics that deals with problems involving complex PDEs in 3 spatial dimensions and in time. It is vital from the beginning to adopt a notation for the equations which is compact yet unambiguous. To make the equations easy to follow, we must use a notation that encapsulates the idea of a tensor as an entity in the own right, rather than a list of scalar components. Additionally, any tensor operation should be perceived as an operation on the entire tensor entity rather than a series of operations on its components.

Consequently, in this book the *tensor notation* is preferred in which any tensor of rank 1 and above, *i.e.* all tensors other than scalars, are represented by letters in bold face, *e.g.* **a**. This actively promotes the concept of a tensor as a entity in its own right since it is denoted by a single symbol, and it is also extremely compact. The potential drawback is that the rank of a bold face symbol is not immediately apparent, although it is clearly not zero. However, in practice this presents no real problem since we are aware of the property each symbol represents and therefore intuitively know its rank, *e.g.* we know velocity  $\mathbf{U}$  is a tensor of rank 1.

A further, more fundamental idea regarding the choice of notation is that the mathematical representation of a tensor should not change depending on our coordinate system, *i.e.* the vector **a** is the same vector irrespective of where we view it from. The tensor notation supports this concept as it implies nothing about the coordinate system. However, other notations, *e.g.*  $a_i$ , expose the individual components of the tensor which naturally implies the choice of coordinate system. The unsatisfactory consequence of this is that the tensor is then represented by a set of values which are not unique — they depend on the coordinate system.

That said, the index notation, introduced in Section 1.2, is adopted from time to time in this book mainly to expand tensor operations into the constituent components. When using the index notation, we adopt the *summation convention* which states that whenever the same letter subscript occurs twice in a term, the that subscript is to be given all values, *i.e.* 1, 2, 3, and the results added together, *e.g.* 

$$a_i b_i = \sum_{i=1}^3 a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$
(1.3)

In the remainder of the book the symbol  $\sum$  is omitted since the repeated subscript indicates the summation.

## **1.3** Algebraic tensor operations

This section describes all the algebraic operations for tensors that are available in Open-FOAM. Let us first review the most simple tensor operations: addition, subtraction, and scalar multiplication and division. Addition and subtraction are both commutative and associative and are only valid between tensors of the same rank. The operations are performed by addition/subtraction of respective components of the tensors, *e.g.* the subtraction of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is

$$\mathbf{a} - \mathbf{b} = a_i - b_i = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \tag{1.4}$$

Multiplication of any tensor  $\mathbf{a}$  by a scalar s is also commutative and associative and is performed by multiplying all the tensor components by the scalar. For example,

$$s\mathbf{a} = sa_i = (sa_1, sa_2, sa_3)$$
 (1.5)

Division between a tensor  $\mathbf{a}$  and a scalar is only relevant when the scalar is the second argument of the operation, *i.e.* 

$$\mathbf{a}/s = a_i/s = (a_1/s, a_2/s, a_3/s) \tag{1.6}$$

Following these operations are a set of more complex products between tensors of rank 1 and above, described in the following Sections.

#### 1.3.1 The inner product

The inner product operates on any two tensors of rank  $r_1$  and  $r_2$  such that the rank of the result  $r = r_1 + r_2 - 2$ . Inner product operations with tensors up to rank 3 are described below:

• The inner product of two vectors **a** and **b** is commutative and produces a scalar  $s = \mathbf{a} \cdot \mathbf{b}$  where

$$s = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{1.7}$$

• The inner product of a tensor  $\mathbf{T}$  and vector  $\mathbf{a}$  produces a vector  $\mathbf{b} = \mathbf{T} \cdot \mathbf{a}$ , represented below as a column array for convenience

$$b_{i} = T_{ij}a_{j} = \begin{pmatrix} T_{11}a_{1} + T_{12}a_{2} + T_{13}a_{3} \\ T_{21}a_{1} + T_{22}a_{2} + T_{23}a_{3} \\ T_{31}a_{1} + T_{32}a_{2} + T_{33}a_{3} \end{pmatrix}$$
(1.8)

It is non-commutative if T is non-symmetric such that  $\mathbf{b} = \mathbf{a} \cdot \mathbf{T} = \mathbf{T}^{\mathrm{T}} \cdot \mathbf{a}$  is

$$b_{i} = a_{j}T_{ji} = \begin{pmatrix} a_{1}T_{11} + a_{2}T_{21} + a_{3}T_{31} \\ a_{1}T_{12} + a_{2}T_{22} + a_{3}T_{32} \\ a_{1}T_{13} + a_{2}T_{23} + a_{3}T_{33} \end{pmatrix}$$
(1.9)

• The inner product of two tensors  $\mathbf{T}$  and  $\mathbf{S}$  produces a tensor  $\mathbf{P} = \mathbf{T} \cdot \mathbf{S}$  whose components are evaluated as:

$$P_{ij} = T_{ik} S_{kj} \tag{1.10}$$

It is non-commutative such that  $\mathbf{T} \cdot \mathbf{S} = (\mathbf{S}^{\mathrm{T}} \cdot \mathbf{T}^{\mathrm{T}})^{\mathrm{T}}$ 

• The inner product of a vector  $\mathbf{a}$  and third rank tensor  $\mathbf{P}$  produces a second rank tensor  $\mathbf{T} = \mathbf{a} \cdot \mathbf{P}$  whose components are

$$T_{ij} = a_k P_{kij} \tag{1.11}$$

Again this is non-commutative so that  $\mathbf{T} = \mathbf{P} \cdot \mathbf{a}$  is

$$T_{ij} = P_{ijk}a_k \tag{1.12}$$

• The inner product of a second rank tensor  $\mathbf{T}$  and third rank tensor  $\mathbf{P}$  produces a third rank tensor  $\mathbf{Q} = \mathbf{T} \cdot \mathbf{P}$  whose components are

$$Q_{ijk} = T_{il}P_{ljk} \tag{1.13}$$

Again this is non-commutative so that  $\mathbf{Q} = \mathbf{P} \cdot \mathbf{T}$  is

$$Q_{ijk} = P_{ijl}T_{lk} \tag{1.14}$$

#### 1.3.2 The double inner product of two tensors

The double inner product of two second-rank tensors  $\mathbf{T}$  and  $\mathbf{S}$  produces a scalar  $s = \mathbf{T} : \mathbf{S}$  which can be evaluated as the sum of the 9 products of the tensor components

$$s = T_{ij}S_{ij} = T_{11}S_{11} + T_{12}S_{12} + T_{13}S_{13} + T_{21}S_{21} + T_{22}S_{22} + T_{23}S_{23} + T_{31}S_{31} + T_{32}S_{32} + T_{33}S_{33}$$

$$(1.15)$$

The double inner product between a second rank tensor  $\mathbf{T}$  and third rank tensor  $\mathbf{P}$  produces a vector  $\mathbf{a} = \mathbf{T} : \mathbf{P}$  with components

$$a_i = T_{jk} P_{jki} \tag{1.16}$$

This is non-commutative so that  $\mathbf{a} = \mathbf{P} \mathbf{:} \mathbf{T}$  is

$$a_i = P_{ijk}T_{jk} \tag{1.17}$$

#### 1.3.3 The triple inner product of two third rank tensors

The triple inner product of two third rank tensors  $\mathbf{P}$  and  $\mathbf{Q}$  produces a scalar  $s = \mathbf{P} \cdot \mathbf{Q}$ which can be evaluated as the sum of the 27 products of the tensor components

$$s = P_{ijk}Q_{ijk} \tag{1.18}$$

#### 1.3.4 The outer product

The outer product operates between vectors and tensors as follows:

• The outer product of two vectors  $\mathbf{a}$  and  $\mathbf{b}$  is non-commutative and produces a tensor  $\mathbf{T} = \mathbf{ab} = (\mathbf{ba})^{\mathrm{T}}$  whose components are evaluated as:

$$T_{ij} = a_i b_j = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$
(1.19)

• An outer product of a vector **a** and second rank tensor **T** produces a third rank tensor  $\mathbf{P} = \mathbf{aT}$  whose components are

$$P_{ijk} = a_i T_{jk} \tag{1.20}$$

This is non-commutative so that  $\mathbf{P} = \mathbf{T} \mathbf{a}$  produces

$$P_{ijk} = T_{ij}a_k \tag{1.21}$$

#### **1.3.5** The cross product of two vectors

The cross product operation is exclusive to vectors only. For two vectors  $\mathbf{a}$  with  $\mathbf{b}$ , it produces a vector  $\mathbf{c} = \mathbf{a} \times \mathbf{b}$  whose components are

$$c_i = e_{ijk}a_jb_k = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$
(1.22)

where the *permutation symbol* is defined by

$$e_{ijk} = \begin{cases} 0 & \text{when any two indices are equal} \\ +1 & \text{when } i, j, k \text{ are an even permutation of } 1, 2, 3 \\ -1 & \text{when } i, j, k \text{ are an odd permutation of } 1, 2, 3 \end{cases}$$
(1.23)

in which the even permutations are 123, 231 and 312 and the odd permutations are 132, 213 and 321.

#### 1.3.6 Other general tensor operations

Some less common tensor operations and terminology used by OpenFOAM are described below.

- **Square** of a tensor is defined as the outer product of the tensor with itself, *e.g.* for a vector **a**, the square  $\mathbf{a}^2 = \mathbf{a}\mathbf{a}$ .
- *n*th power of a tensor is evaluated by *n* outer products of the tensor, *e.g.* for a vector **a**, the 3rd power  $\mathbf{a}^3 = \mathbf{a}\mathbf{a}\mathbf{a}$ .
- **Magnitude squared** of a tensor is the *r*th inner product of the tensor of rank *r* with itself, to produce a scalar. For example, for a second rank tensor  $\mathbf{T}$ ,  $|\mathbf{T}|^2 = \mathbf{T} : \mathbf{T}$ .
- **Magnitude** is the square root of the magnitude squared, *e.g.* for a tensor  $\mathbf{T}$ ,  $|\mathbf{T}| = \sqrt{\mathbf{T}:\mathbf{T}}$ . Vectors of unit magnitude are referred to as *unit vectors*.
- **Component maximum** is the component of the tensor with greatest value, inclusive of sign, *i.e.* not the largest magnitude.
- Component minimum is the component of the tensor with smallest value.
- Component average is the mean of all components of a tensor.
- Scale As the name suggests, the scale function is a tool for scaling the components of one tensor by the components of another tensor of the same rank. It is evaluated as the product of corresponding components of 2 tensors, *e.g.*, scaling vector **a** by vector **b** would produce vector **c** whose components are

$$c_i = \text{scale}(\mathbf{a}, \mathbf{b}) = (a_1 b_1, a_2 b_2, a_3 b_3)$$
 (1.24)

#### **1.3.7** Geometric transformation and the identity tensor

A second rank tensor **T** is strictly defined as a linear vector function, i.e. it is a function which associates an argument vector **a** to another vector **b** by the inner product  $\mathbf{b} = \mathbf{T} \cdot \mathbf{a}$ . The components of **T** can be chosen to perform a specific geometric transformation of a tensor from the x, y, z coordinate system to a new coordinate system  $x^*, y^*, z^*$ ; **T** is then referred to as the *transformation tensor*. While a scalar remains unchanged under a transformation, the vector **a** is transformed to  $\mathbf{a}^*$  by

$$\mathbf{a}^* = \mathbf{T} \cdot \mathbf{a} \tag{1.25}$$

A second rank tensor  ${\bf S}$  is transformed to  ${\bf S}^*$  according to

$$\mathbf{S}^* = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{T}^{\mathrm{T}} \tag{1.26}$$

The *identity tensor* I is defined by the requirement that it transforms another tensor onto itself. For all vectors **a** 

$$\mathbf{a} = \mathbf{I} \cdot \mathbf{a} \tag{1.27}$$

and therefore

$$\mathbf{I} = \delta_{ij} = \begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{pmatrix}$$
(1.28)

where  $\delta_{ij}$  is known as the *Kronecker delta* symbol.

### 1.3.8 Useful tensor identities

Several identities are listed below which can be verified by under the assumption that all the relevant derivatives exist and are continuous. The identities are expressed for scalar s and vector **a**.

 $\nabla \cdot (\nabla \times \mathbf{a}) \equiv 0$   $\nabla \times (\nabla s) \equiv \mathbf{0}$   $\nabla \cdot (s\mathbf{a}) \equiv s \nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla s$   $\nabla \times (s\mathbf{a}) \equiv s \nabla \times \mathbf{a} + \nabla s \times \mathbf{a}$   $\nabla (\mathbf{a} \cdot \mathbf{b}) \equiv \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) + (\mathbf{a} \cdot \nabla)\mathbf{b} + (\mathbf{b} \cdot \nabla)\mathbf{a}$   $\nabla \cdot (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$   $\nabla \times (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{a} (\nabla \cdot \mathbf{b}) - \mathbf{b} (\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b}$   $\nabla \times (\nabla \times \mathbf{a}) \equiv \nabla (\nabla \cdot \mathbf{a}) - \nabla^{2}\mathbf{a}$   $(\nabla \times \mathbf{a}) \times \mathbf{a} \equiv \mathbf{a} \cdot (\nabla \mathbf{a}) - \nabla (\mathbf{a} \cdot \mathbf{a})$ (1.29)

It is sometimes useful to know the  $e - \delta$  identity to help to manipulate equations in index notation:

$$e_{ijk}e_{irs} = \delta_{jr}\delta_{ks} - \delta_{js}\delta_{kr} \tag{1.30}$$

#### 1.3.9 Operations exclusive to tensors of rank 2

There are several operations that manipulate the components of tensors of rank 2 that are listed below:

**Transpose** of a tensor  $\mathbf{T} = T_{ij}$  is  $\mathbf{T}^{\mathrm{T}} = T_{ji}$  as described in Equation 1.2.

Symmetric and skew (antisymmetric) tensors As discussed in section 1.2, a tensor is said to be symmetric if its components are symmetric about the diagonal, i.e.  $\mathbf{T} = \mathbf{T}^{\mathrm{T}}$ . A skew or antisymmetric tensor has  $\mathbf{T} = -\mathbf{T}^{\mathrm{T}}$  which intuitively implies that  $T_{11} = T_{22} = T_{33} = 0$ . Every second order tensor can be decomposed into symmetric and skew parts by

$$\mathbf{T} = \underbrace{\frac{1}{2}(\mathbf{T} + \mathbf{T}^{\mathrm{T}})}_{symmetric} + \underbrace{\frac{1}{2}(\mathbf{T} - \mathbf{T}^{\mathrm{T}})}_{skew} = \operatorname{symm} \mathbf{T} + \operatorname{skew} \mathbf{T}$$
(1.31)

**Trace** The trace of a tensor **T** is a scalar, evaluated by summing the diagonal components

$$\operatorname{tr} \mathbf{T} = T_{11} + T_{22} + T_{33} \tag{1.32}$$

 $\mathbf{Diagonal}\ \mathrm{returns}\ \mathrm{a}\ \mathrm{vector}\ \mathrm{whose}\ \mathrm{components}\ \mathrm{are}\ \mathrm{the}\ \mathrm{diagonal}\ \mathrm{components}\ \mathrm{of}\ \mathrm{the}\ \mathrm{second}\ \mathrm{rank}\ \mathrm{tensor}\ \mathbf{T}$ 

diag 
$$\mathbf{T} = (T_{11}, T_{22}, T_{33})$$
 (1.33)

**Deviatoric and hydrostatic tensors** Every second rank tensor  $\mathbf{T}$  can be decomposed into a deviatoric component, for which tr  $\mathbf{T} = 0$  and a hydrostatic component of the form  $\mathbf{T} = s\mathbf{I}$  where s is a scalar. Every second rank tensor can be decomposed into deviatoric and hydrostatic parts as follows:

$$\mathbf{T} = \underbrace{\mathbf{T} - \frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{deviatoric} + \underbrace{\frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{hydrostatic} = \operatorname{dev} \mathbf{T} + \operatorname{hyd} \mathbf{T}$$
(1.34)

Determinant The determinant of a second rank tensor is evaluated by

$$\det \mathbf{T} = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = \begin{aligned} T_{11}(T_{22}T_{33} - T_{23}T_{32}) - \\ T_{12}(T_{21}T_{33} - T_{23}T_{31}) + \\ T_{13}(T_{21}T_{32} - T_{22}T_{31}) \end{aligned}$$
(1.35)
$$= \frac{1}{6} e_{ijk} e_{pqr} T_{ip} T_{jq} T_{kr}$$

**Cofactors** The *minors* of a tensor are evaluated for each component by deleting the row and column in which the component is situated and evaluating the resulting entries as a  $2 \times 2$  determinant. For example, the minor of  $T_{12}$  is

$$\begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = \begin{vmatrix} T_{21} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = T_{21}T_{33} - T_{23}T_{31}$$
(1.36)

The cofactors are *signed minors* where each minor is component is given a sign based on the rule

+ve if 
$$i + j$$
 is even  
-ve if  $i + j$  is odd (1.37)

The cofactors of  ${\bf T}$  can be evaluated as

$$\operatorname{cof} \mathbf{T} = \frac{1}{2} e_{jkr} e_{ist} T_{sk} T_{tr} \tag{1.38}$$

**Inverse** The inverse of a tensor can be evaluated as

$$\operatorname{inv} \mathbf{T} = \frac{\operatorname{cof} \mathbf{T}^{\mathrm{T}}}{\det \mathbf{T}} \tag{1.39}$$

Hodge dual of a tensor is a vector whose components are

$$*\mathbf{T} = (T_{23}, -T_{13}, T_{12}) \tag{1.40}$$

#### **1.3.10** Operations exclusive to scalars

OpenFOAM supports most of the well known functions that operate on scalars, *e.g.* square root, exponential, logarithm, sine, cosine *etc...*, a list of which can be found in Table 1.2. There are 3 additional functions defined within OpenFOAM that are described below:

**Sign** of a scalar s is

$$sgn(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ -1 & \text{if } s < 0. \end{cases}$$
(1.41)

**Positive** of a scalar s is

$$pos(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ 0 & \text{if } s < 0. \end{cases}$$
(1.42)

**Limit** of a scalar s by the scalar n

$$\operatorname{limit}(s,n) = \begin{cases} s & \text{if } s < n, \\ 0 & \text{if } s \ge n. \end{cases}$$
(1.43)

## 1.4 OpenFOAM tensor classes

OpenFOAM contains a C++ class library primitive that contains the classes for the tensor mathematics described so far. The basic tensor classes that are available as standard in OpenFOAM are listed in Table 1.1. The Table also lists the functions that allow the user to access individual components of a tensor, known as access functions.

Rank	Common name	Basic class	Access functions
0	Scalar	scalar	
1	Vector	vector	x(),y(),z()
2	Tensor	tensor	xx(), xy(), xz()

Table 1.1: Basic tensor classes in OpenFOAM

We can declare the tensor

$$\mathbf{T} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \tag{1.44}$$

in OpenFOAM by the line:

tensor T(1, 2, 3, 4, 5, 6, 7, 8, 9);

We can then access the component  $T_{13}$ , or  $T_{xz}$  using the xz() access function. For instance the code

outputs to the screen:

Txz = 3

#### 1.4.1 Algebraic tensor operations in OpenFOAM

The algebraic operations described in Section 1.3 are all available to the OpenFOAM tensor classes using syntax which closely mimics the notation used in written mathematics. Some functions are represented solely by descriptive functions, e.g.symm(), but others can also be executed using symbolic operators, e.g.\*. All functions are listed in Table 1.2.

Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Addition		$\mathbf{a} + \mathbf{b}$	a + b
Subtraction		$\mathbf{a} - \mathbf{b}$	a - b
Scalar multiplication		sa	s * a
Scalar division		$\mathbf{a}/s$	a/s
Outer product	rank $\mathbf{a}, \mathbf{b} >= 1$	ab	a * b
Inner product	rank $\mathbf{a}, \mathbf{b} >= 1$	a•b	a & b
Double inner product	rank $\mathbf{a}, \mathbf{b} \ge 2$	a:b	a && b
Cross product	rank $\mathbf{a}, \mathbf{b} = 1$	$\mathbf{a} \times \mathbf{b}$	a î b
Square		$\mathbf{a}^2$	sqr(a)
		Co	ntinued on next page

Continued from previous page			
Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Magnitude squared		$ \mathbf{a} ^2$	magSqr(a)
Magnitude		a	mag(a)
Power	n = 0, 1,, 4	$\mathbf{a}^n$	pow(a,n)
Component average	i=1,,N	$\overline{a_i}$	cmptAv(a)
Component maximum	$i = 1, \dots, N$	$\max(a_i)$	max(a)
Component minimum	i = 1,, N	$\min(a_i)$	min(a)
Scale		$\text{scale}(\mathbf{a}, \mathbf{b})$	<pre>scale(a,b)</pre>
Geometric transformation	transforms $\mathbf{a}$ u	using tensor $\mathbf{T}$	<pre>transform(T,a)</pre>

# Operations exclusive to tensors of rank 2

Transpose	$\mathbf{T}^{ ext{T}}$	T.T()
Diagonal	$\operatorname{diag} \mathbf{T}$	diag(T)
Trace	$\operatorname{tr} \mathbf{T}$	tr(T)
Deviatoric component	$\operatorname{dev} \mathbf{T}$	dev(T)
Symmetric component	$\operatorname{symm} \mathbf{T}$	symm(T)
Skew-symmetric component	$\operatorname{skew} \mathbf{T}$	skew(T)
Determinant	$\det \mathbf{T}$	det(T)
Cofactors	$\operatorname{cof}\mathbf{T}$	cof(T)
Inverse	$\operatorname{inv} \mathbf{T}$	inv(T)
Hodge dual	* T	*T

## Operations exclusive to scalars

operations exclusive to a	scalar 5		
Sign (boolean)		$\operatorname{sgn}(s)$	sign(s)
Positive (boolean)		$s \ge 0$	pos(s)
Negative (boolean)		s < 0	neg(s)
Limit	n scalar	limit(s, n)	limit(s,n)
Square root		$\sqrt{s}$	sqrt(s)
Exponential		$\exp s$	exp(s)
Natural logarithm		$\ln s$	log(s)
Base 10 logarithm		$\log_{10} s$	log10(s)
Sine		$\sin s$	sin(s)
Cosine		$\cos s$	cos(s)
Tangent		$\tan s$	tan(s)
Arc sine		$\operatorname{asin} s$	asin(s)
Arc cosine		$a\cos s$	acos(s)
Arc tangent		$\operatorname{atan} s$	atan(s)
Hyperbolic sine		$\sinh s$	sinh(s)
Hyperbolic cosine		$\cosh s$	cosh(s)
Hyperbolic tangent		$\tanh s$	tanh(s)
Hyperbolic arc sine		$\operatorname{asinh} s$	asinh(s)
Hyperbolic arc cosine		$\operatorname{acosh} s$	acosh(s)
Hyperbolic arc tangent		$\operatorname{atanh} s$	atanh(s)
Error function		$\operatorname{erf} s$	erf(s)
Complement error function		$\operatorname{erfc} s$	erfc(s)
Logarithm gamma function		$\ln \Gamma s$	lgamma(s)
Type 1 Bessel function of or	der 0	$J_0 s$	j0(s)
Type 1 Bessel function of or	der 1	$J_1 s$	j1(s)
		C	ontinued on next page

Continued from previous page					
Operation	Comment	Mathematical	Description		
		Description	in OpenFOAM		
Type 2 Bessel function of order	$Y_0 s$	y0(s)			
Type 2 Bessel function of order	$Y_1 s$	y1(s)			
<b>a</b> , <b>b</b> are tensors of arbitrary rank unless otherwise stated					

a, b are tensors of arbitrary rank unless otherwise sta

s is a scalar, N is the number of tensor components

Table 1.2: Algebraic tensor operations in OpenFOAM

## 1.5 Dimensional units

In continuum mechanics, properties are represented in some chosen units, e.g. mass in kilograms (kg), volume in cubic metres (m<sup>3</sup>), pressure in Pascals (kg m s<sup>-2</sup>). Algebraic operations must be performed on these properties using consistent units of measurement; in particular, addition, subtraction and equality are only physically meaningful for properties of the same dimensional units. As a safeguard against implementing a meaningless operation, OpenFOAM encourages the user to attach dimensional units to any tensor and will then perform dimension checking of any tensor operation.

Units are defined using the dimensionSet class, e.g.

dimensionSet pressureDims(1, -1, -2, 0, 0, 0, 0);

No.	Property	Unit	Symbol
1	Mass	kilogram	k
2	Length	metre	m
3	Time	second	S
4	Temperature	Kelvin	Κ
5	Quantity	moles	mol
6	Current	ampere	А
7	Luminous intensity	candela	cd

where each of the values corresponds to the power of each of the S.I. base units of measurement listed in Table 1.3. The line of code declares pressureDims to be the dimensionSet for pressure kg m s<sup>-2</sup> since the first entry in the pressureDims array, 1, corresponds to k<sup>1</sup>, the second entry, -1, corresponds to m<sup>-1</sup> etc.. A tensor with units is defined using the dimensioned<Type> template class, the <Type> being scalar, vector, tensor, etc.. The dimensioned<Type> stores a variable name of class word, the value <Type> and a dimensionSet

```
dimensionedTensor sigma
  (
     "sigma",
     dimensionSet(1, -1, -2, 0, 0, 0, 0),
     tensor(1e6,0,0,0,1e6,0,0,0,1e6),
   );
```

creates a tensor with correct dimensions of pressure, or stress

$$\sigma = \begin{pmatrix} 10^6 & 0 & 0\\ 0 & 10^6 & 0\\ 0 & 0 & 10^6 \end{pmatrix}$$
(1.45)

# Chapter 2

# **Discretisation procedures**

So far we have dealt with algebra of tensors at a point. The PDEs we wish to solve involve derivatives of tensors with respect to time and space. We therefore need to extend our description to a *tensor field*, *i.e.* a tensor that varies across time and spatial domains. In this Chapter we will first present a mathematical description of all the differential operators we may encounter. We will then show how a tensor field is constructed in OpenFOAM and how the derivatives of these fields are discretised into a set of algebraic equations.

### 2.1 Differential operators

Before defining the spatial derivatives we first introduce the nabla vector operator  $\nabla$ , represented in index notation as  $\partial_i$ :

$$\nabla \equiv \partial_i \equiv \frac{\partial}{\partial x_i} \equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$$
(2.1)

The nabla operator is a useful notation that obeys the following rules:

- it operates on the tensors to its right and the conventional rules of a derivative of a product, *e.g.*  $\partial_i ab = (\partial_i a) b + a (\partial_i b);$
- otherwise the nabla operator behaves like any other vector in an algebraic operation.

#### 2.1.1 Gradient

If a scalar field s is defined and continuously differentiable then the gradient of  $s, \nabla s$  is a vector field

$$\nabla s = \partial_i s = \left(\frac{\partial s}{\partial x_1}, \frac{\partial s}{\partial x_2}, \frac{\partial s}{\partial x_3}\right) \tag{2.2}$$

The gradient can operate on any tensor field to produce a tensor field that is one rank higher. For example, the gradient of a vector field **a** is a second rank tensor field

$$\nabla \mathbf{a} = \partial_i a_j = \begin{pmatrix} \partial a_1 / \partial x_1 & \partial a_2 / \partial x_1 & \partial a_3 / \partial x_1 \\ \partial a_1 / \partial x_2 & \partial a_2 / \partial x_2 & \partial a_3 / \partial x_2 \\ \partial a_1 / \partial x_3 & \partial a_2 / \partial x_3 & \partial a_3 / \partial x_3 \end{pmatrix}$$
(2.3)

#### 2.1.2 Divergence

If a vector field  ${\bf a}$  is defined and continuously differentiable then the divergence of  ${\bf a}$  is a scalar field

$$\nabla \cdot \mathbf{a} = \partial_i a_i = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3}$$
(2.4)

The divergence can operate on any tensor field of rank 1 and above to produce a tensor that is one rank lower. For example the divergence of a second rank tensor field  $\mathbf{T}$  is a vector field (expanding the vector as a column array for convenience)

$$\nabla \cdot \mathbf{T} = \partial_i T_{ij} = \begin{pmatrix} \partial T_{11} / \partial x_1 + \partial T_{12} / \partial x_1 + \partial T_{13} / \partial x_1 \\ \partial T_{21} / \partial x_2 + \partial T_{22} / \partial x_2 + \partial T_{23} / \partial x_2 \\ \partial T_{31} / \partial x_3 + \partial T_{32} / \partial x_3 + \partial T_{33} / \partial x_3 \end{pmatrix}$$
(2.5)

#### 2.1.3 Curl

If a vector field **a** is defined and continuously differentiable then the curl of **a**,  $\nabla \times \mathbf{a}$  is a vector field

$$\nabla \times \mathbf{a} = e_{ijk} \partial_j a_k = \left( \frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3}, \frac{\partial a_1}{\partial x_3} - \frac{\partial a_3}{\partial x_1}, \frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2} \right)$$
(2.6)

The curl is related to the gradient by

$$\nabla \times \mathbf{a} = 2 \,(* \, \mathrm{skew} \, \nabla \mathbf{a}) \tag{2.7}$$

#### 2.1.4 Laplacian

The Laplacian is an operation that can be defined mathematically by a combination of the divergence and gradient operators by  $\nabla^2 \equiv \nabla \cdot \nabla$ . However, the Laplacian should be considered as a single operation that transforms a tensor field into another tensor field of the same rank, rather than a combination of two operations, one which raises the rank by 1 and one which reduces the rank by 1.

In fact, the Laplacian is best defined as a *scalar operator*, just as we defined nabla as a vector operator, by

$$\nabla^2 \equiv \partial^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$
(2.8)

For example, the Laplacian of a scalar field s is the scalar field

$$\nabla^2 s = \partial^2 s = \frac{\partial^2 s}{\partial x_1^2} + \frac{\partial^2 s}{\partial x_2^2} + \frac{\partial^2 s}{\partial x_3^2}$$
(2.9)

#### 2.1.5 Temporal derivative

There is more than one definition of temporal, or time, derivative of a tensor. To describe the temporal derivatives we must first recall that the tensor relates to a property of a volume of material that may be moving. If we track an infinitesimally small volume of material, or particle, as it moves and observe the change in the tensorial property  $\phi$  in time, we have the *total*, or *material* time derivative denoted by

$$\frac{D\phi}{Dt} = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t}$$
(2.10)

However in continuum mechanics, particularly fluid mechanics, we often observe the change of a  $\phi$  in time at a fixed point in space as different particles move across that point. This change at a point in space is termed the *spatial* time derivative which is denoted by  $\partial/\partial t$  and is related to the material derivative by:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{U} \cdot \nabla\phi \tag{2.11}$$

where U is the velocity field of property  $\phi$ . The second term on the right is known as the convective rate of change of  $\phi$ .

## 2.2 Overview of discretisation

The term discretisation means approximation of a problem into discrete quantities. The FV method and others, such as the finite element and finite difference methods, all discretise the problem as follows:

- **Spatial discretisation** Defining the solution domain by a set of points that fill and bound a region of space when connected;
- **Temporal discretisation** (For transient problems) dividing the time domain into into a finite number of time intervals, or steps;
- **Equation discretisation** Generating a system of algebraic equations in terms of discrete quantities defined at specific locations in the domain, from the PDEs that characterise the problem.

## 2.2.1 OpenFOAM lists and fields

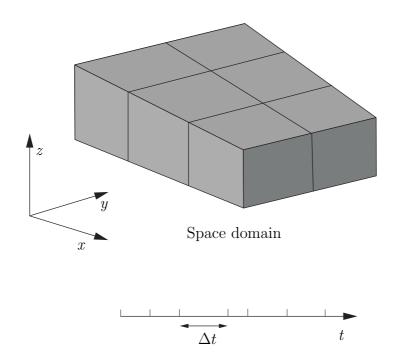
OpenFOAM frequently needs to store sets of data and perform functions, such as mathematical operations, on the data. OpenFOAM therefore provides an array template class List<Type>, making it possible to create a list of any object of class Type that inherits the functions of the Type. For example a List of vector is List<vector>.

Lists of the tensor classes are defined as standard in OpenFOAM by the template class Field<Type>. For better code legibility, all instances of Field<Type>, *e.g.*Field<vector>, are renamed using typedef declarations as scalarField, vectorField, tensorField, symmTensor-Field, tensorThirdField and symmTensorThirdField. Algebraic operations can be performed between Fields subject to obvious restrictions such as the fields having the same number of elements. OpenFOAM also supports operations between a field and single tensor, *e.g.* all values of a Field U can be multiplied by the scalar 2 with the operation U = 2.0 \* U.

# 2.3 Discretisation of the solution domain

Discretisation of the solution domain is shown in Figure 2.1. The space domain is discretised into computational mesh on which the PDEs are subsequently discretised. Discretisation of time, if required, is simple: it is broken into a set of time steps  $\Delta t$  that may change during a numerical simulation, perhaps depending on some condition calculated during the simulation.

On a more detailed level, discretisation of space requires the subdivision of the domain into a number of cells, or control volumes. The cells are contiguous, *i.e.* they do not overlap one another and completely fill the domain. A typical cell is shown in Figure 2.2.



Time domain Figure 2.1: Discretisation of the solution domain

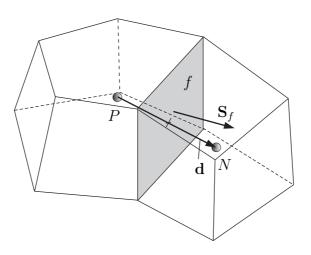


Figure 2.2: Parameters in finite volume discretisation

Dependent variables and other properties are principally stored at the cell centroid P although they may be stored on faces or vertices. The cell is bounded by a set of flat faces, given the generic label f. In OpenFOAM there is no limitation on the number of faces bounding each cell, nor any restriction on the alignment of each face. This kind of mesh is often referred to as "arbitrarily unstructured" to differentiate it from meshes in which the cell faces have a prescribed alignment, typically with the coordinate axes. Codes with arbitrarily unstructured meshes offer greater freedom in mesh generation and manipulation in particular when the geometry of the domain is complex or changes over time.

Whilst most properties are defined at the cell centroids, some are defined at cell faces. There are two types of cell face.

- **Internal faces** Those faces that connect two cells (and it can never be more than two). For each internal face, OpenFOAM designates one adjoining cell to be the face *owner* and the other to be the *neighbour*;
- **Boundary faces** Those belonging to one cell since they coincide with the boundary of the domain. These faces simply have an owner cell.

## 2.3.1 Defining a mesh in OpenFOAM

There are different levels of mesh description in OpenFOAM, beginning with the most basic mesh class, named **polyMesh** since it is based on polyhedra. A **polyMesh** is constructed using the minimum information required to define the mesh geometry described below and presented in Figure 2.3:

- **Points** A list of cell vertex point coordinate vectors, *i.e.* a vectorField, that is renamed pointField using a typedef declaration;
- **Faces** A list of cell faces List<face>, or faceList, where the face class is defined by a list of vertex numbers, corresponding to the pointField;
- **Cells** a list of cells List<cell>, or cellList, where the cell class is defined by a list of face numbers, corresponding to the faceList described previously.
- **Boundary** a polyBoundaryMesh decomposed into a list of patches, polyPatchList representing different regions of the boundary. The boundary is subdivided in this manner to allow different boundary conditions to be specified on different patches during a solution. All the faces of any polyPatch are stored as a single block of the faceList, so that its faces can be easily accessed using the slice class which stores references to the first and last face of the block. Each polyPatch is then constructed from
  - $\bullet$  a slice;
  - a word to assign it a name.

FV discretisation uses specific data that is derived from the mesh geometry stored in polyMesh. OpenFOAM therefore extends the polyMesh class to fvMesh which stores the additional data needed for FV discretisation. fvMesh is constructed from polyMesh and stores the data in Table 2.1 which can be updated during runtime in cases where the mesh moves, is refined *etc.*.

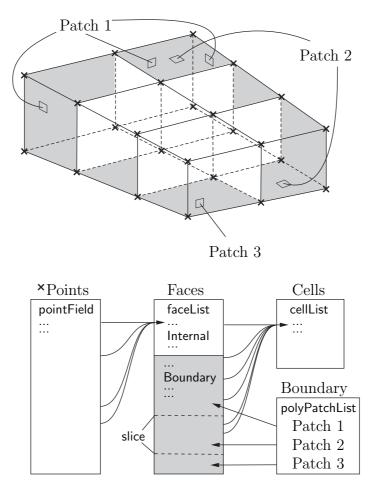


Figure 2.3: Schematic of the basic mesh description used in OpenFOAM

## 2.3.2 Defining a geometricField in OpenFOAM

So far we can define a field, *i.e.* a list of tensors, and a mesh. These can be combined to define a tensor field relating to discrete points in our domain, specified in OpenFOAM by the template class geometricField<Type>. The Field values are separated into those defined within the internal region of the domain, *e.g.* at the cell centres, and those defined on the domain boundary, *e.g.* on the boundary faces. The geometricField<Type> stores the following information:

Internal field This is simply a Field<Type>, described in Section 2.2.1;

- **BoundaryField** This is a GeometricBoundaryField, in which a Field is defined for the faces of each patch and a Field is defined for the patches of the boundary. This is then a field of fields, stored within an object of the FieldField<Type> class. A reference to the fvBoundaryMesh is also stored [\*\*].
- **Mesh** A reference to an fvMesh, with some additional detail as to the whether the field is defined at cell centres, faces, *etc.*.

Dimensions A dimensionSet, described in Section 4.2.6.

**Old values** Discretisation of time derivatives requires field data from previous time steps. The geometricField<Type> will store references to stored fields from the previous, or old, time step and its previous, or old-old, time step where necessary.

Class	Description	Symbol	Access function
volScalarField	Cell volumes	V	V()
surfaceVectorField	Face area vectors	$\mathbf{S}_{f}$	Sf()
surfaceScalarField	Face area magnitudes	$ \mathbf{S}_{f} $	magSf()
volVectorField	Cell centres	С	C()
surfaceVectorField	Face centres	$\mathbf{C}_{f}$	Cf()
surfaceScalarField	Face motion fluxes **	$\phi_g$	phi()

Table 2.1: fvMesh stored data.

**Previous iteration values** The iterative solution procedures can use under-relaxation which requires access to data from the previous iteration. Again, if required, geo-metricField<Type> stores a reference to the data from the previous iteration.

As discussed in Section 2.3, we principally define a property at the cell centres but quite often it is stored at the cell faces and on occasion it is defined on cell vertices. The geometricField<Type> is renamed using typedef declarations to indicate where the field variable is defined as follows:

volField<Type> A field defined at cell centres;

surfaceField<Type> A field defined on cell faces;

```
pointField<Type> A field defined on cell vertices.
```

These typedef field classes of geometricField<Type>are illustrated in Figure 2.4. A geometricField<Type> inherits all the tensor algebra of Field<Type> and has all operations subjected to dimension checking using the dimensionSet. It can also be subjected to the FV discretisation procedures described in the following Section. The class structure used to build geometricField<Type> is shown in Figure 2.5<sup>1</sup>.

## 2.4 Equation discretisation

Equation discretisation converts the PDEs into a set of algebraic equations that are commonly expressed in matrix form as:

 $[A] [x] = [b] \tag{2.12}$ 

where [A] is a square matrix, [x] is the column vector of dependent variable and [b] is the source vector. The description of [x] and [b] as 'vectors' comes from matrix terminology rather than being a precise description of what they truly are: a list of values defined at locations in the geometry, *i.e.* a geometricField<Type>, or more specifically a volField<Type> when using FV discretisation.

[A] is a list of coefficients of a set of algebraic equations, and cannot be described as a geometricField<Type>. It is therefore given a class of its own: fvMatrix. fvMatrix<Type> is created through discretisation of a geometric<Type>Field and therefore inherits the <Type>. It supports many of the standard algebraic matrix operations of addition +, subtraction - and multiplication \*.

Each term in a PDE is represented individually in OpenFOAM code using the classes of static functions finiteVolumeMethod and finiteVolumeCalculus, abbreviated by a typedef

<sup>&</sup>lt;sup>1</sup>The diagram is not an exact description of the class hierarchy, rather a representation of the general structure leading from some primitive classes to geometric < Type > Field.

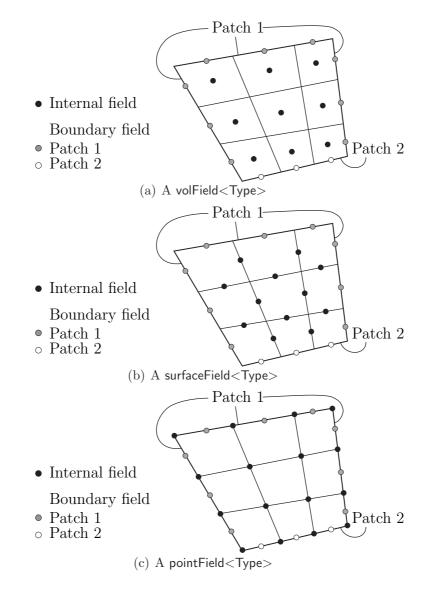


Figure 2.4: Types of geometricField<Type> defined on a mesh with 2 boundary patches (in 2 dimensions for simplicity)

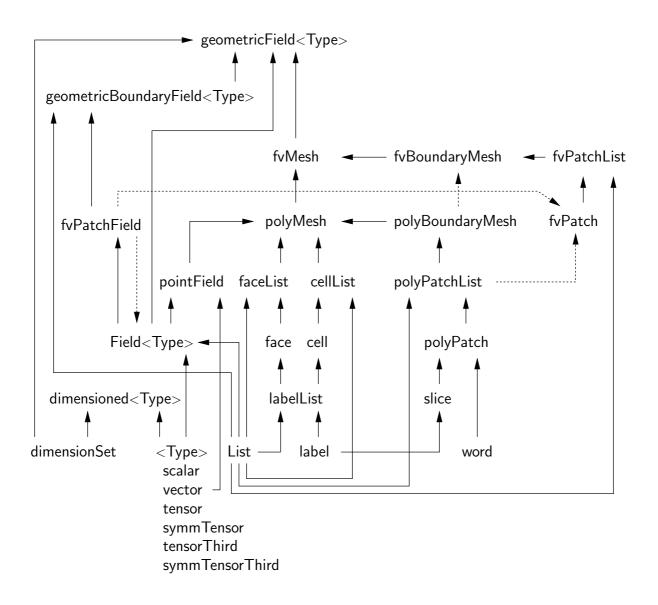


Figure 2.5: Basic class structure leading to geometricField<Type>

to fvm and fvc respectively. fvm and fvc contain static functions, representing differential operators, *e.g.*  $\nabla^2$ ,  $\nabla \cdot$  and  $\partial/\partial t$ , that discretise geometricField<Type>s. The purpose of defining these functions within two classes, fvm and fvc, rather than one, is to distinguish:

- functions of fvm that calculate implicit derivatives of and return an fvMatrix<Type>
- some functions of fvc that calculate explicit derivatives and other explicit calculations, returning a geometricField<Type>.

Figure 2.6 shows a geometricField<Type> defined on a mesh with 2 boundary patches and illustrates the explicit operations merely transform one field to another and drawn in 2D for simplicity.

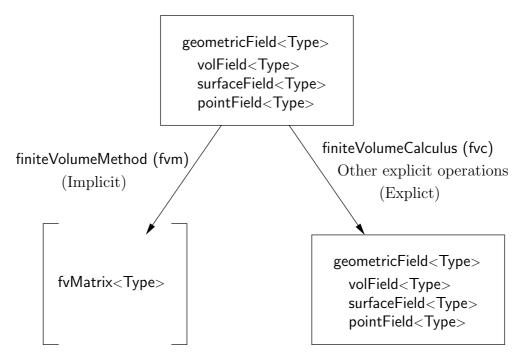


Figure 2.6: A geometricField<Type> and its operators

Table 2.2 lists the main functions that are available in fvm and fvc to discretise terms that may be found in a PDE. FV discretisation of each term is formulated by first integrating the term over a cell volume V. Most spatial derivative terms are then converted to integrals over the cell surface S bounding the volume using Gauss's theorem

$$\int_{V} \nabla \star \phi \, dV = \int_{S} d\mathbf{S} \star \phi \tag{2.13}$$

where **S** is the surface area vector,  $\phi$  can represent any tensor field and the star notation  $\star$  is used to represent any tensor product, *i.e.* inner, outer and cross and the respective derivatives: divergence  $\nabla \cdot \phi$ , gradient  $\nabla \phi$  and  $\nabla \times \phi$ . Volume and surface integrals are then linearised using appropriate schemes which are described for each term in the following Sections. Some terms are always discretised using one scheme, a selection of schemes is offered in OpenFOAM for the discretisation of other terms. The choice of scheme is either made by a direct specification within the code or it can be read from an input file at job run-time and stored within an fvSchemes class object.

Term description	Implicit /	Text	fvm::/fvc:: functions
	Explicit	expression	
Laplacian	Imp/Exp	$\nabla^2 \phi$	laplacian(phi)
		$\nabla \bullet \Gamma \nabla \phi$	laplacian(Gamma, phi)
Time derivative	Imp/Exp	$\frac{\partial \phi}{\partial t}$	ddt(phi)
		$\frac{\partial \rho \phi}{\partial t}$	ddt(rho,phi)
Second time derivative	Imp/Exp	$\frac{\partial}{\partial t} \left( \rho \frac{\partial \phi}{\partial t} \right)$	d2dt2(rho, phi)
Convection	Imp/Exp	$ abla ullet (\psi)$	div(psi,scheme)*
		$ abla ullet (\psi \phi)$	div(psi, phi, word)*
			div(psi, phi)
Divergence	Exp	$ abla ullet \chi$	div(chi)
Gradient	Exp	$ abla \chi$	grad(chi)
		$ abla \phi$	gGrad(phi)
			lsGrad(phi)
			snGrad(phi)
			<pre>snGradCorrection(phi)</pre>
Grad-grad squared	Exp	$  abla abla\phi ^2$	sqrGradGrad(phi)
Curl	Exp	$\nabla  imes \phi$	curl(phi)
Source	Imp	$ ho\phi$	Sp(rho,phi)
	$\mathrm{Imp}/\mathrm{Exp}^{\dagger}$		SuSp(rho,phi)

ffvm::SuSp source is discretised implicit or explicit depending on the sign of rho. †An explicit source can be introduced simply as a vol<Type>Field, e.g.rho\*phi. Function arguments can be of the following classes:

phi: vol<Type>Field

 $\label{eq:Gamma:scalar} \begin{array}{l} \mbox{Gamma: scalar volScalarField, surfaceScalarField, volTensorField, surfaceTensorField. } \\ \mbox{rho: scalar, volScalarField} \end{array}$ 

psi: surfaceScalarField.

chi: surface<Type>Field, vol<Type>Field.

Table 2.2: Discretisation of PDE terms in OpenFOAM

#### 2.4.1 The Laplacian term

The Laplacian term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\Gamma \nabla \phi) \, dV = \int_{S} d\mathbf{S} \cdot (\Gamma \nabla \phi) = \sum_{f} \Gamma_{f} \mathbf{S}_{f} \cdot (\nabla \phi)_{f}$$
(2.14)

The face gradient discretisation is implicit when the length vector **d** between the centre of the cell of interest P and the centre of a neighbouring cell N is orthogonal to the face plane, *i.e.* parallel to  $\mathbf{S}_{f}$ :

$$\mathbf{S}_{f} \bullet (\nabla \phi)_{f} = |S_{f}| \frac{\phi_{N} - \phi_{P}}{|\mathbf{d}|}$$
(2.15)

In the case of non-orthogonal meshes, an additional explicit term is introduced which is evaluated by interpolating cell centre gradients, themselves calculated by central differencing cell centre values.

#### 2.4.2 The convection term

The convection term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\rho \mathbf{U}\phi) \, dV = \int_{S} d\mathbf{S} \cdot (\rho \mathbf{U}\phi) = \sum_{f} \mathbf{S}_{f} \cdot (\rho \mathbf{U})_{f} \phi_{f} = \sum_{f} F \phi_{f}$$
(2.16)

The face field  $\phi_f$  can be evaluated using a variety of schemes:

Central differencing (CD) is second-order accurate but unbounded

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N \tag{2.17}$$

where  $f_x \equiv \overline{fN}/\overline{PN}$  where  $\overline{fN}$  is the distance between f and cell centre N and  $\overline{PN}$  is the distance between cell centres P and N.

**Upwind differencing (UD)** determines  $\phi_f$  from the direction of flow and is bounded at the expense of accuracy

$$\phi_f = \begin{cases} \phi_P & \text{for } F \ge 0\\ \phi_N & \text{for } F < 0 \end{cases}$$
(2.18)

**Blended differencing (BD)** schemes combine UD and CD in an attempt to preserve boundedness with reasonable accuracy,

$$\phi_f = (1 - \gamma) \left(\phi_f\right)_{UD} + \gamma \left(\phi_f\right)_{CD} \tag{2.19}$$

OpenFOAM has several implementations of the Gamma differencing scheme to select the blending coefficient  $\gamma$  but it offers other well-known schemes such as van Leer, SUPERBEE, MINMOD *etc.*.

# 2.4.3 First time derivative

The first time derivative  $\partial/\partial t$  is integrated over a control volume as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \, dV \tag{2.20}$$

The term is discretised by simple differencing in time using:

**new values**  $\phi^n \equiv \phi(t + \Delta t)$  at the time step we are solving for;

old values  $\phi^o \equiv \phi(t)$  that were stored from the previous time step;

**old-old values**  $\phi^{oo} \equiv \phi(t - \Delta t)$  stored from a time step previous to the last.

One of two discretisation schemes can be declared using the timeScheme keyword in the appropriate input file, described in detail in section 4.4 of the User Guide.

Euler implicit scheme, timeScheme EulerImplicit, that is first order accurate in time:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{\left(\rho_{P} \phi_{P} V\right)^{n} - \left(\rho_{P} \phi_{P} V\right)^{o}}{\Delta t} \tag{2.21}$$

Backward differencing scheme, timeScheme BackwardDifferencing, that is second order accurate in time by storing the old-old values and therefore with a larger overhead in data storage than EulerImplicit:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{3 \left(\rho_P \phi_P V\right)^n - 4 \left(\rho_P \phi_P V\right)^o + \left(\rho_P \phi_P V\right)^{oo}}{2\Delta t} \tag{2.22}$$

#### 2.4.4 Second time derivative

The second time derivative is integrated over a control volume and linearised as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \frac{\partial \phi}{\partial t} \, dV = \frac{\left(\rho_{P} \phi_{P} V\right)^{n} - 2\left(\rho_{P} \phi_{P} V\right)^{o} + \left(\rho_{P} \phi_{P} V\right)^{oo}}{\Delta t^{2}} \tag{2.23}$$

It is first order accurate in time.

#### 2.4.5 Divergence

The divergence term described in this Section is strictly an explicit term that is distinguished from the convection term of Section 2.4.2, *i.e.* in that it is not the divergence of the product of a velocity and dependent variable. The term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot \phi \, dV = \int_{S} d\mathbf{S} \cdot \phi = \sum_{f} \mathbf{S}_{f} \cdot \phi_{f}$$
(2.24)

The fvc::div function can take as its argument either a surface<Type>Field, in which case  $\phi_f$  is specified directly, or a vol<Type>Field which is interpolated to the face by central differencing as described in Section 2.4.10:

# 2.4.6 Gradient

The gradient term is an explicit term that can be evaluated in a variety of ways. The scheme can be evaluated either by selecting the particular grad function relevant to the discretisation scheme, *e.g.*fvc::gGrad, fvc::lsGrad *etc.*, or by using the fvc::grad function combined with the appropriate timeScheme keyword in an input file

Gauss integration is invoked using the fvc::grad function with timeScheme Gauss or directly using the fvc::gGrad function. The discretisation is performed using the standard method of applying Gauss's theorem to the volume integral:

$$\int_{V} \nabla \phi \, dV = \int_{S} d\mathbf{S} \, \phi = \sum_{f} \mathbf{S}_{f} \phi_{f} \tag{2.25}$$

As with the fvc::div function, the Gaussian integration fvc::grad function can take either a surfaceField<Type> or a volField<Type> as an argument.

Least squares method is based on the following idea:

- 1. a value at point P can be extrapolated to neighbouring point N using the gradient at P;
- 2. the extrapolated value at N can be compared to the actual value at N, the difference being the error;
- 3. if we now minimise the sum of the square of weighted errors at all neighbours of P with the respect to the gradient, then the gradient should be a good approximation.

Least squares is invoked using the fvc::grad function with timeScheme leastSquares or directly using the fvc::lsGrad function. The discretisation is performed as by first calculating the tensor **G** at every point *P* by summing over neighbours *N*:

$$\mathbf{G} = \sum_{N} w_{N}^{2} \mathbf{d} \mathbf{d}$$
(2.26)

where **d** is the vector from *P* to *N* and the weighting function  $w_N = 1/|\mathbf{d}|$ . The gradient is then evaluated as:

$$(\nabla \phi)_P = \sum_N w_N^2 \mathbf{G}^{-1} \cdot \mathbf{d} \left( \phi_N - \phi_P \right)$$
(2.27)

**Surface normal gradient** The gradient normal to a surface  $\mathbf{n}_f \cdot (\nabla \phi)_f$  can be evaluated at cell faces using the scheme

$$(\nabla\phi)_f = \frac{\phi_N - \phi_P}{|\mathbf{d}|} \tag{2.28}$$

This gradient is called by the function fvc::snGrad and returns a surfaceField<Type>. The scheme is directly analogous to that evaluated for the Laplacian discretisation scheme in Section 2.4.1, and in the same manner, a correction can be introduced to improve the accuracy of this face gradient in the case of non-orthogonal meshes. This correction is called using the function fvc::snGradCorrection [Check\*\*].

# 2.4.7 Grad-grad squared

The grad-grad squared term is evaluated by: taking the gradient of the field; taking the gradient of the resulting gradient field; and then calculating the magnitude squared of the result. The mathematical expression for grad-grad squared of  $\phi$  is  $|\nabla (\nabla \phi)|^2$ .

# 2.4.8 Curl

The curl is evaluated from the gradient term described in Section 2.4.6. First, the gradient is discretised and then the curl is evaluated using the relationship from Equation 2.7, repeated here for convenience

 $\nabla \times \phi = 2 \, * (\operatorname{skew} \nabla \phi)$ 

## 2.4.9 Source terms

Source terms can be specified in 3 ways

**Explicit** Every explicit term is a volField<Type>. Hence, an explicit source term can be incorporated into an equation simply as a field of values. For example if we wished to solve Poisson's equation  $\nabla^2 \phi = f$ , we would define phi and f as volScalarField and then do

Implicit An implicit source term is integrated over a control volume and linearised by

$$\int_{V} \rho \phi \, dV = \rho_P V_P \phi_P \tag{2.29}$$

Implicit/Explicit The implicit source term changes the coefficient of the diagonal of the matrix. Depending on the sign of the coefficient and matrix terms, this will either increase or decrease diagonal dominance of the matrix. Decreasing the diagonal dominance could cause instability during iterative solution of the matrix equation. Therefore OpenFOAM provides a mixed source discretisation procedure that is implicit when the coefficients that are greater than zero, and explicit for the coefficients less than zero. In mathematical terms the matrix coefficient for node Pis  $V_P \max(\rho_P, 0)$  and the source term is  $V_P \phi_P \min(\rho_P, 0)$ .

# 2.4.10 Other explicit discretisation schemes

There are some other discretisation procedures that convert  $volField{<}Type{>}s$  into  $surface{<}Type{>}Fields$  and visa versa.

- Surface integral fvc::surfaceIntegrate performs a summation of surface<Type>Field face values bounding each cell and dividing by the cell volume, *i.e.*  $(\sum_f \phi_f)/V_P$ . It returns a volField<Type>.
- Surface sum fvc::surfaceSum performs a summation of surface<Type>Field face values bounding each cell, *i.e.*  $\sum_{f} \phi_{f}$  returning a volField<Type>.
- Average fvc::average produces an area weighted average of surface<Type>Field face values, *i.e.*  $(\sum_{f} S_{f} \phi_{f}) / \sum_{f} S_{f}$ , and returns a volField<Type>.

#### Reconstruct

Face interpolate The geometric<Type>Field function faceInterpolate() interpolates
volField<Type> cell centre values to cell faces using central differencing, returning a
surface<Type>Field.

# 2.5 Temporal discretisation

Although we have described the discretisation of temporal derivatives in Sections 2.4.3 and 2.4.4, we need to consider how to treat the spatial derivatives in a transient problem. If we denote all the spatial terms as  $\mathcal{A}\phi$  where  $\mathcal{A}$  is any spatial operator, *e.g.* Laplacian, then we can express a transient PDE in integral form as

$$\int_{t}^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V} \rho \phi \, dV + \int_{V} \mathcal{A}\phi \, dV \right] \, dt = 0 \tag{2.30}$$

Using the Euler implicit method of Equation 2.21, the first term can be expressed as

$$\int_{t}^{t+\Delta t} \left[ \frac{\partial}{\partial t} \int_{V} \rho \phi \, dV \right] dt = \int_{t}^{t+\Delta t} \frac{\left(\rho_{P} \phi_{P} V\right)^{n} - \left(\rho_{P} \phi_{P} V\right)^{o}}{\Delta t} \, dt$$

$$= \frac{\left(\rho_{P} \phi_{P} V\right)^{n} - \left(\rho_{P} \phi_{P} V\right)^{o}}{\Delta t} \Delta t$$
(2.31)

The second term can be expressed as

$$\int_{t}^{t+\Delta t} \left[ \int_{V} \mathcal{A}\phi \ dV \right] \ dt = \int_{t}^{t+\Delta t} \mathcal{A}^{*}\phi \ dt \tag{2.32}$$

where  $\mathcal{A}^*$  represents the spatial discretisation of  $\mathcal{A}$ . The time integral can be discretised in three ways:

**Euler implicit** uses implicit discretisation of the spatial terms, thereby taking current values  $\phi^n$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^{*} \phi \, dt = \mathcal{A}^{*} \phi^{n} \Delta t \tag{2.33}$$

It is first order accurate in time, guarantees boundedness and is unconditionally stable.

**Explicit** uses explicit discretisation of the spatial terms, thereby taking old values  $\phi^{o}$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^{*}\phi \, dt = \mathcal{A}^{*}\phi^{o}\Delta t \tag{2.34}$$

It is first order accurate in time and is unstable if the Courant number Co is greater than 1. The Courant number is defined as

$$Co = \frac{\mathbf{U}_f \cdot \mathbf{d}}{|\mathbf{d}|^2 \Delta t} \tag{2.35}$$

where  $\mathbf{U}_{f}$  is a characteristic velocity, *e.g.* velocity of a wave front, velocity of flow.

**Crank Nicholson** uses the trapezoid rule to discretise the spatial terms, thereby taking a mean of current values  $\phi^n$  and old values  $\phi^o$ .

$$\int_{t}^{t+\Delta t} \mathcal{A}^{*} \phi \, dt = \mathcal{A}^{*} \left( \frac{\phi^{n} + \phi^{o}}{2} \right) \Delta t \tag{2.36}$$

It is second order accurate in time, is unconditionally stable but does not guarantee boundedness.

# 2.5.1 Treatment of temporal discretisation in OpenFOAM

At present the treatment of the temporal discretisation is controlled by the implementation of the spatial derivatives in the PDE we wish to solve. For example, let us say we wish to solve a transient diffusion equation

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi \tag{2.37}$$

An Euler implicit implementation of this would read

```
solve(fvm::ddt(phi) == kappa*fvm::laplacian(phi))
```

where we use the  $\mathsf{fvm}$  class to discretise the Laplacian term implicitly. An explicit implementation would read

solve(fvm::ddt(phi) == kappa\*fvc::laplacian(phi))

where we now use the fvc class to discretise the Laplacian term explicitly. The Crank Nicholson scheme can be implemented by the mean of implicit and explicit terms:

```
solve
  (
  fvm::ddt(phi)
  ==
  kappa*0.5*(fvm::laplacian(phi) + fvc::laplacian(phi))
  )
```

# 2.6 Boundary Conditions

Boundary conditions are required to complete the problem we wish to solve. We therefore need to specify boundary conditions on all our boundary faces. Boundary conditions can be divided into 2 types:

- **Dirichlet** prescribes the value of the dependent variable on the boundary and is therefore termed 'fixed value' in this guide;
- **Neumann** prescribes the gradient of the variable normal to the boundary and is therefore termed 'fixed gradient' in this guide.

When we perform discretisation of terms that include the sum over faces  $\sum_{f}$ , we need to consider what happens when one of the faces is a boundary face.

**Fixed value** We specify a fixed value at the boundary  $\phi_b$ 

- We can simply substitute  $\phi_b$  in cases where the discretisation requires the value on a boundary face  $\phi_f$ , *e.g.* in the convection term in Equation 2.16.
- In terms where the face gradient  $(\nabla \phi)_f$  is required, *e.g.* Laplacian, it is calculated using the boundary face value and cell centre value,

$$\mathbf{S}_{f} \cdot (\nabla \phi)_{f} = |S_{f}| \, \frac{\phi_{b} - \phi_{P}}{|\mathbf{d}|} \tag{2.38}$$

**Fixed gradient** The fixed gradient boundary condition  $g_b$  is a specification on inner product of the gradient and unit normal to the boundary, or

$$g_b = \left(\frac{\mathbf{S}}{|\mathbf{S}|} \cdot \nabla \phi\right)_f \tag{2.39}$$

• When discretisation requires the value on a boundary face  $\phi_f$  we must interpolate the cell centre value to the boundary by

$$\phi_f = \phi_P + \mathbf{d} \cdot (\nabla \phi)_f$$
  
=  $\phi_P + |\mathbf{d}| g_b$  (2.40)

•  $\phi_b$  can be directly substituted in cases where the discretisation requires the face gradient to be evaluated,

$$\mathbf{S}_f \bullet (\nabla \phi)_f = |S_f| \, g_b \tag{2.41}$$

#### 2.6.1 Physical boundary conditions

The specification of boundary conditions is usually an engineer's interpretation of the true behaviour. Real boundary conditions are generally defined by some physical attributes rather than the numerical description as described of the previous Section. In incompressible fluid flow there are the following physical boundaries

- **Inlet** The velocity field at the inlet is supplied and, for consistency, the boundary condition on pressure is zero gradient.
- **Outlet** The pressure field at the outlet is supplied and a zero gradient boundary condition on velocity is specified.
- **No-slip impermeable wall** The velocity of the fluid is equal to that of the wall itself, *i.e.* a fixed value condition can be specified. The pressure is specified zero gradient since the flux through the wall is zero.

In a problem whose solution domain and boundary conditions are symmetric about a plane, we only need to model half the domain to one side of the symmetry plane. The boundary condition on the plane must be specified according to

**Symmetry plane** The symmetry plane condition specifies the component of the gradient normal to the plane should be zero. [Check\*\*]

# Chapter 3

# Examples of the use of OpenFOAM

In this section we shall describe several test cases supplied with the OpenFOAM distribution. The intention is to provide example cases, including those in the tutorials in chapter 2 of the User Guide, for every standard solver. The examples are designed to introduce certain tools and features of OpenFOAM, *e.g.* within pre-/post-processing, numerical schemes, algorithms. They also provide a means for validation of solvers although that is not their principal function.

Each example contains a description of the problem: the geometry, initial and boundary conditions, a brief description of the equations being solved, models used, and physical properties required. The solution domain is selected which may be a portion of the original geometry, *e.g.* if we introduce symmetry planes. The method of meshing, usually **blockMesh**, is specified; of course the user can simply view the mesh since every example is distributed with the *polyMesh* directory containing the data files that describe the mesh.

The examples coexist with the tutorials in the *tutorials* subdirectory of the OpenFOAM installation. They are organised into a set of subdirectories by solver, *e.g.* all the icoFoam cases are stored within a subdirectory *icoFoam*. Before running a particular example, the user is urged to copy it into their user account. We recommend that the user stores all OpenFOAM cases in a directory we recommend that the tutorials are copied into a directory *\$FOAM\_RUN*. If this directory structure has not yet been created in the user's account, it can be created with

```
mkdir -p $FOAM_RUN
```

The tutorials can then be copied into this directory with

```
cp -r $FOAM_TUTORIALS/* $FOAM_RUN
```

# **3.1** Flow around a cylinder

In this example we shall investigate potential flow around a cylinder using potentialFoam. This example introduces the following OpenFOAM features:

- non-orthogonal meshes;
- generating an analytical solution to a problem in OpenFOAM.

# 3.1.1 Problem specification

The problem is defined as follows:

**Solution domain** The domain is 2 dimensional and consists of a square domain with a cylinder collocated with the centre of the square as shown in Figure 3.1.

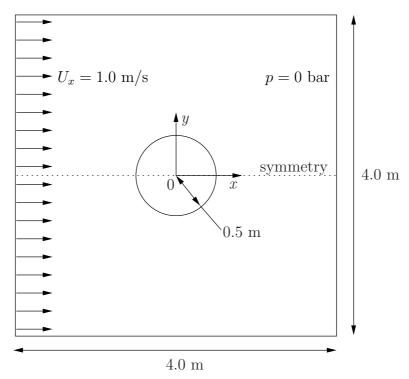


Figure 3.1: Geometry of flow round a cylinder

#### Governing equations

• Mass continuity for an incompressible fluid

$$\nabla \cdot \mathbf{U} = 0 \tag{3.1}$$

• Pressure equation for an incompressible, irrotational fluid assuming steadystate conditions

$$\nabla^2 p = 0 \tag{3.2}$$

#### Boundary conditions

- Inlet (left) with fixed velocity  $\mathbf{U} = (1, 0, 0)$  m/s.
- Outlet (right) with a fixed pressure p = 0 Pa.
- No-slip wall (bottom);
- Symmetry plane (top).
- **Initial conditions** U = 0 m/s, p = 0 Pa required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.
- **Solver name** potentialFoam: a potential flow code, *i.e.* assumes the flow is incompressible, steady, irrotational, inviscid and it ignores gravity.

Case name cylinder case located in the  $FOAM_TUTORIALS/potentialFoam$  directory.

# 3.1.2 Note on potentialFoam

potentialFoam is a useful solver to validate OpenFOAM since the assumptions of potential flow are such that an analytical solution exists for cases whose geometries are relatively simple. In this example of flow around a cylinder an analytical solution exists with which we can compare our numerical solution. potentialFoam can also be run more like a utility to provide a (reasonably) conservative initial **U** field for a problem. When running certain cases, this can useful for avoiding instabilities due to the initial field being unstable. In short, potentialFoam creates a conservative field from a non-conservative initial field supplied by the user.

# 3.1.3 Mesh generation

Mesh generation using blockMesh has been described in tutorials in the User Guide. In this case, the mesh consists of 10 blocks as shown in Figure 3.2. Remember that all

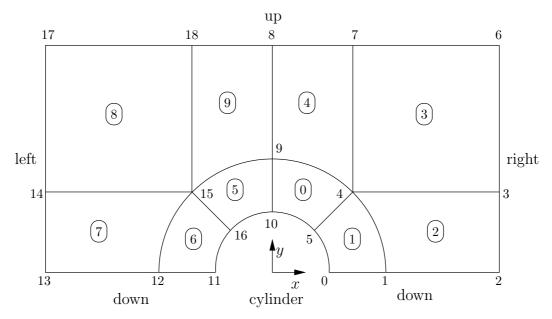


Figure 3.2: Blocks in cylinder geometry

meshes are treated as 3 dimensional in OpenFOAM. If we wish to solve a 2 dimensional problem, we must describe a 3 dimensional mesh that is only one cell thick in the third direction that is not solved. In Figure 3.2 we show only the back plane of the geometry, along z = -0.5, in which the vertex numbers are numbered 0-18. The other 19 vertices in the front plane, z = +0.5, are numbered in the same order as the back plane, as shown in the mesh description file below:

```
--*- C++
2
                    F
                      ield
                                         OpenFOAM: The Open Source CFD Toolbox
3
                    O peration
                                         Version:
                                                    2.0.0
4
                                                     www.OpenFOAM.com
\mathbf{5}
                    A nd
                                         Web:
                    M anipulation
6
7
     FoamFile
8
9
                        2.0;
10
          version
          format
                        ascii;
11
                        dictionary
12
          class
                        blockMeshDict
13
          object
     }
14
15
16
     convertToMeters 1;
17
```

```
18
     vertices #codeStream
19
20
          codeInclude
21
         #{
22
              #include "pointField.H"
23
24
         #};
25
         code
26
         #{
27
              pointField points(19);
28
              points[0]
                           = point(0.5, 0, -0.5);
29
                           = point(1, 0, -0.5);
              points[1]
30
              points[2]
                           = point(2, 0, -0.5);
31
                           = point(2, 0.707107, -0.5);
              points[3]
32
                           = point(0.707107, 0.707107, -0.5);
              points[4]
33
                           = point(0.353553, 0.353553, -0.5);
34
              points[5]
              points[6]
                           = point(2, 2, -0.5);
35
              points[7]
                           = point(0.707107, 2, -0.5);
36
                           = point(0, 2, -0.5);
= point(0, 1, -0.5);
              points[8]
37
              points[9]
38
              points[10]
                           = point(0, 0.5, -0.5)
39
              points[11] = point(-0.5, 0, -0.5);
40
              points[12] = point(-1, 0, -0.5);
41
              points[13] = point(-2, 0, -0.5);
42
              points[14]
                           = point(-2, 0.707107,
                                                     -0.5);
43
                           = point(-0.707107, 0.707107, -0.5);
44
              points[15]
              points[16] = point(-0.353553, 0.353553, -0.5);
45
              points[17] = point(-2, 2, -0.5);
46
              points[18] = point(-0.707107, 2, -0.5);
47
48
              // Duplicate z points
49
              label sz = points.size();
50
              points.setSize(2*sz);
51
              for (label i = 0; i < sz; i++)
52
53
              ł
                   const point& pt = points[i];
54
                   points[i+sz] = point(pt.x(), pt.y(), -pt.z());
55
              }
56
57
              os
                  << points;
58
         #};
59
    };
60
61
62
    blocks
63
64
         hex (5 4 9 10 24 23 28 29) (10 10 1) simpleGrading (1 1 1)
65
         hex (0 1 4 5 19 20 23 24) (10 10 1) simpleGrading (1 1 1)
66
         hex (1 2 3 4 20 21 22 23) (20 10 1) simpleGrading (1 1 1)
67
         hex (4 3 6 7 23 22 25 26) (20 20 1) simpleGrading (1 1 1)
68
         hex (9 4 7 8 28 23 26 27) (10 20 1) simpleGrading (1 1 1)
69
         hex (15 16 10 9 34 35 29 28) (10 10 1) simpleGrading (1 1 1)
70
         hex (12 11 16 15 31 30 35 34) (10 10 1) simpleGrading (1 1 1)
71
         hex (13 12 15 14 32 31 34 33) (20 10 1) simpleGrading (1 1 1)
72
         hex (14 15 18 17 33 34 37 36) (20 20 1) simpleGrading (1 1 1)
73
         hex (15 9 8 18 34 28 27 37) (10 20 1) simpleGrading (1 1 1)
74
    );
75
76
     edges
77
78
     (
         arc 0 5 (0.469846 0.17101 -0.5)
79
         arc 5 10 (0.17101 0.469846 -0.5)
arc 1 4 (0.939693 0.34202 -0.5)
arc 4 9 (0.34202 0.939693 -0.5)
80
81
82
         arc 19 24 (0.469846 0.17101 0.5)
arc 24 29 (0.17101 0.469846 0.5)
                      (0.469846 0.17101 0.5)
83
84
         arc 20 23
arc 23 28
                     (0.939693 \ 0.34202 \ 0.5)
(0.34202 \ 0.939693 \ 0.5)
(-0.469846 \ 0.17101 \ -0.5)
(-0.17101 \ 0.469846 \ -0.5)
85
86
         arc 11 16
87
88
         arc 16 10
                    (-0.939693 0.34202 -0.5)
(-0.34202 0.939693 -0.5)
         arc 12
89
                  15
         arc 15 9
90
         arc 30 35
arc 35 29
                      (-0.469846 0.17101 0.5)
91
                      (-0.17101 0.469846 0.5)
92
                      (-0.939693 0.34202
              31
                  34
                                            0.5)
93
         \operatorname{arc}
         arc 34 28
                     (-0.34202 0.939693 0.5)
94
    );
95
96
```

97 boundary ( 98 down 99 100 type symmetryPlane; 101 faces 102 103 ( (0 1 20 19) (1 2 21 20) (12 11 30 31) (13 12 31 32) 104 105 106 107 ); 108 } 109 right 110 { 111 112 type patch; faces 113 114 ( (2 3 22 21) 6 25 22) 115116 ); 117 } 118 119up 120 type symmetryPlane; 121faces 122 ( 123 (7 8 27 26) (6 7 26 25) (8 18 37 27) (18 17 36 37) 124125126 127); 128 } left 129 130 131 132 type patch; faces 133 134  $(14 \ 13 \ 32 \ 33) \\ (17 \ 14 \ 33 \ 36)$ 135 136 ); 137 138 } cylinder 139 140 type symmetryPlane; 141 faces 142 ( 143 (10 5 24 29) 144(5 0 19 24) (16 10 29 35) (11 16 35 30) 145146 147 ); 148 } 149); 150151mergePatchPairs 152153( ); 154155156

#### 3.1.4 Boundary conditions and initial fields

Using FoamX or editing case files by hand, set the boundary conditions in accordance with the problem description in Figure 3.1, *i.e.* the left boundary should be an Inlet, the right boundary should be an Outlet and the down and cylinder boundaries should be symmetryPlane. The top boundary conditions is chosen so that we can make the most genuine comparison with our analytical solution which uses the assumption that the domain is infinite in the y direction. The result is that the normal gradient of **U** is small along a plane coinciding with our boundary. We therefore impose the condition that the normal component is zero, *i.e.* specify the boundary as a symmetryPlane, thereby ensuring that the comparison with the analytical is reasonable.

#### 3.1.5 Running the case

No fluid properties need be specified in this problem since the flow is assumed to be incompressible and inviscid. In the *system* subdirectory, the *controlDict* specifies the control parameters for the run. Note that since we assume steady flow, we only run for 1 time step:

```
---*- C++ -*-----
 1
2
                    F ield
                                        OpenFOAM: The Open Source CFD Toolbox
3
                    O peration
                                        Version:
                                                   2.0.0
4
                                                    www.OpenFOAM.com
                    {\tt A} \ {\tt nd}
                                        Web:
5
                    M anipulation
6
           ١١.
                                      7
    FoamFile
 8
9
     {
                       2.0;
ascii;
         version
10
11
         format
                       dictionary;
"system";
         class
12
13
         location
                       controlDict;
14
         object
15
    11
          *
            *
               *
                 *
                           * * *
                                                                                             * //
16
17
    application
                       potentialFoam;
18
19
    startFrom
                       startTime;
20
21
    startTime
                       0;
22
23
                       endTime;
24
    stopAt
25
    endTime
26
                       1;
27
    deltaT
28
                       1;
29
    writeControl
                       timeStep;
30
31
    writeInterval
32
                       1;
33
    purgeWrite
                       0;
34
35
36
    writeFormat
                       ascii;
37
    writePrecision 6;
38
39
    writeCompression off;
40
41
    timeFormat
                       general;
42
43
    timePrecision
                       6;
44
45
46
    runTimeModifiable true;
47
    functions
48
49
         difference
50
51
              // Load the library containing the 'coded' functionObject
52
              functionObjectLibs ("libutilityFunctionObjects.so");
53
              type coded;
54
              // Name of on-the-fly generated functionObject
55
              redirectType error;
56
              code
57
              #{
58
                   // Lookup U
59
                   Info<< "Looking up field U\n" << endl;</pre>
60
                   const volVectorField& U = mesh().lookupObject<volVectorField>("U");
61
62
                   Info<< "Reading inlet velocity uInfX\n" << endl;</pre>
63
64
                   dimensionedScalar uInfX
65
66
                        "uInfx"
67
                       dimensionSet(0, 1, -1, 0, 0),
U.boundaryField()[3][0].x()
68
69
70
                   Info << "U at inlet = " << uInfX.value() << " m/s" << endl;</pre>
71
72
                   dimensionedScalar radius
73
74
                        "radius",
75
```

```
dimensionSet(0, 1, 0, 0, 0),
mag(U.mesh().boundary()[4].Cf()[0])
76
77
                  ):
78
79
                  Info << "Cylinder radius = " << radius.value() << " m" << endl;</pre>
80
81
                  volVectorField UA
82
83
                       IOobject
84
85
                       (
                           "UA"
86
                           mesh().time().timeName(),
87
                           U.mesh(),
IOobject::NO_READ
88
89
                           IOobject::AUTO_WRÍTE
90
91
                       )
U
92
                  );
93
94
                  Info<< "\nEvaluating analytical solution" << endl;</pre>
95
96
                  const volVectorField& centres = UA.mesh().C();
97
                  volScalarField magCentres(mag(centres));
98
                  volScalarField theta(acos((centres & vector(1,0,0))/magCentres));
99
100
                  volVectorField cs2theta
101
102
                      cos(2*theta)*vector(1,0,0)
sin(2*theta)*vector(0,1,0)
103
104
                  ):
105
106
                  UA = uInfX*(dimensionedVector(vector(1,0,0))
107
                     - pow((radius/magCentres),2)*cs2theta);
108
109
                  // Force writing of UA (since time has not changed)
110
                  UA.write();
111
112
                  volScalarField error("error", mag(U-UA)/mag(UA));
113
114
                  Info<<"Writing relative error in U to " << error.objectPath()</pre>
115
                       << endl;
116
117
                  error.write();
118
             #};
119
         }
120
     }
121
122
123
     124
```

potentialFoam executes an iterative loop around the pressure equation which it solves in order that explicit terms relating to non-orthogonal correction in the Laplacian term may be updated in successive iterations. The number of iterations around the pressure equation is controlled by the nNonOrthogonalCorrectors keyword in *controlDict*. In the first instance we can set nNonOrthogonalCorrectors to 0 so that no loops are performed, *i.e.* the pressure equation is solved once, and there is no non-orthogonal correction. The solution is shown in Figure 3.3(a) (at t = 1, when the steady-state simulation is complete). We expect the solution to show smooth streamlines passing across the domain as in the analytical solution in Figure 3.3(c), yet there is clearly some error in the regions where there is high non-orthogonality in the mesh, *e.g.* at the join of blocks 0, 1 and 3. The case can be run a second time with some non-orthogonal correction by setting nNonOrthogonalCorrectors to 3. The solution shows smooth streamlines with no significant error due to non-orthogonality as shown in Figure 3.3(b).

# 3.2 Steady turbulent flow over a backward-facing step

In this example we shall investigate steady turbulent flow over a backward-facing step. The problem description is taken from one used by Pitz and Daily in an experimental investigation [\*\*] against which the computed solution can be compared. This example introduces the following OpenFOAM features for the first time:

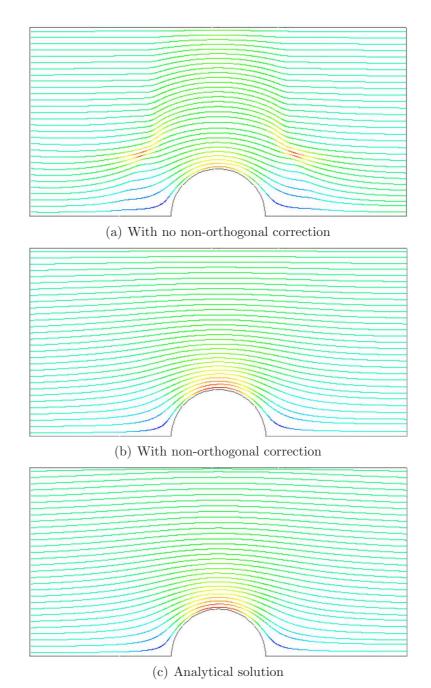


Figure 3.3: Streamlines of potential flow

- generation of a mesh using blockMesh using full mesh grading capability;
- steady turbulent flow.

## 3.2.1 Problem specification

The problem is defined as follows:

**Solution domain** The domain is 2 dimensional, consisting of a short inlet, a backward-facing step and converging nozzle at outlet as shown in Figure 3.4.



Figure 3.4: Geometry of backward-facing step

#### Governing equations

• Mass continuity for incompressible flow

$$\nabla \cdot \mathbf{U} = 0 \tag{3.3}$$

• Steady flow momentum equation

$$\nabla \cdot (\mathbf{U}\mathbf{U}) + \nabla \cdot \mathbf{R} = -\nabla p \tag{3.4}$$

where p is kinematic pressure and (in slightly over-simplistic terms)  $\mathbf{R} = \nu_{eff} \nabla \mathbf{U}$  is the viscous stress term with an effective kinematic viscosity  $\nu_{eff}$ , calculated from selected transport and turbulence models.

**Initial conditions** U = 0 m/s, p = 0 Pa — required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.

#### Boundary conditions

- Inlet (left) with fixed velocity  $\mathbf{U} = (10, 0, 0) \text{ m/s};$
- Outlet (right) with fixed pressure p = 0 Pa;
- No-slip walls on other boundaries.

#### Transport properties

• Kinematic viscosity of air  $\nu = \mu/\rho = 18.1 \times 10^{-6}/1.293 = 14.0 \ \mu m^2/s$ 

#### Turbulence model

- Standard  $k \epsilon$ ;
- Coefficients:  $C_{\mu} = 0.09; C_1 = 1.44; C_2 = 1.92; \alpha_k = 1; \alpha_{\epsilon} = 0.76923.$

Solver name simpleFoam: an implementation for steady incompressible flow.

Case name pitzDaily, located in the \$FOAM\_TUTORIALS/simpleFoam directory.

The problem is solved using simpleFoam, so-called as it is an implementation for steady flow using the SIMPLE algorithm [\*\*]. The solver has full access to all the turbulence models in the incompressibleTurbulenceModels library and the non-Newtonian models in-compressibleTransportModels library of the standard OpenFOAM release.

#### 3.2.2 Mesh generation

We expect that the flow in this problem is reasonably complex and an optimum solution will require grading of the mesh. In general, the regions of highest shear are particularly critical, requiring a finer mesh than in the regions of low shear. We can anticipate where high shear will occur by considering what the solution might be in advance of any calculation. At the inlet we have strong uniform flow in the x direction and, as it passes over the step, it generates shear on the fluid below, generating a vortex in the bottom half of the domain. The regions of high shear will therefore be close to the centreline of the domain and close to the walls.

The domain is subdivided into 12 blocks as shown in Figure 3.5.

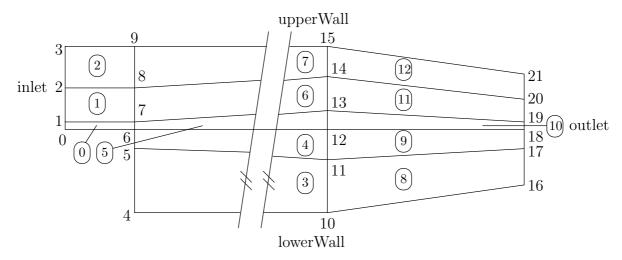


Figure 3.5: Blocks in backward-facing step

The mesh is 3 dimensional, as always in OpenFOAM, so in Figure 3.5 we are viewing the back plane along z = -0.5. The full set of vertices and blocks are given in the mesh description file below:

-\*- C++ 2 F ield OpenFOAM: The Open Source CFD Toolbox 3 Version: 2.0.0 O peration 4 www.OpenFOAM.com A ndWeb: 56 M anipulation 7 FoamFile 8 9 version 2.0; 10 format ascii; 11 dictionary 12 class 13 object ockMeshDict: 14 \* \* // 15

16 17 18 19 20 21	convertToMeters 0.001; vertices ( (-20.6 0 -0.5)
22 23 24 25 26 27 28 29 30 31	$ \begin{pmatrix} -20.6 & 3 & -0.5 \end{pmatrix} \\ (-20.6 & 12.7 & -0.5) \\ (-20.6 & 25.4 & -0.5) \\ (0 & -25.4 & -0.5) \\ (0 & -5 & -0.5) \\ (0 & 0 & -0.5) \\ (0 & 3 & -0.5) \\ (0 & 3 & -0.5) \\ (0 & 25.4 & -0.5) \\ (206 & -25.4 & -0.5) \\ \end{pmatrix} $
32 33 34 35 36 37 38 39 40	$\begin{array}{c} (206 -8.5 -0.5) \\ (206 0 -0.5) \\ (206 6.5 -0.5) \\ (206 17 -0.5) \\ (206 25.4 -0.5) \\ (290 -16.6 -0.5) \\ (290 -6.3 -0.5) \\ (290 0 -0.5) \\ (290 4.5 -0.5) \end{array}$
41 42 43 44 45 46 47 48 49 50	$\begin{array}{c} (290 \ 11 \ -0.5) \\ (290 \ 16.6 \ -0.5) \\ (-20.6 \ 0.5) \\ (-20.6 \ 3 \ 0.5) \\ (-20.6 \ 12.7 \ 0.5) \\ (-20.6 \ 25.4 \ 0.5) \\ (0 \ -25.4 \ 0.5) \\ (0 \ -5 \ 0.5) \\ (0 \ -5 \ 0.5) \\ (0 \ 0 \ 0.5) \\ (0 \ 3 \ 0.5) \end{array}$
51 52 53 54 55 56 57 58 59 60 61	$ \begin{array}{c} (0 & 12.7 & 0.5) \\ (0 & 25.4 & 0.5) \\ (206 & -25.4 & 0.5) \\ (206 & -8.5 & 0.5) \\ (206 & 0 & 0.5) \\ (206 & 6.5 & 0.5) \\ (206 & 17 & 0.5) \\ (206 & 25.4 & 0.5) \\ (290 & -16.6 & 0.5) \\ (290 & -6.3 & 0.5) \\ (290 & 0 & 0.5) \end{array} $
62 63 64	(290 4.5 0.5) (290 11 0.5) (290 16.6 0.5)
65 66 67	); blocks
68 69 70 71 72 73 74 75 76 77 78 79 80 81	<pre>(     hex (0 6 7 1 22 28 29 23) (18 7 1) simpleGrading (0.5 1.8 1)     hex (1 7 8 2 23 29 30 24) (18 10 1) simpleGrading (0.5 4 1)     hex (2 8 9 3 24 30 31 25) (18 13 1) simpleGrading (0.5 0.25 1)     hex (4 10 11 5 26 32 33 27) (180 18 1) simpleGrading (4 1 1)     hex (5 11 12 6 27 33 34 28) (180 9 1) edgeGrading (4 4 4 4 0.5 1 1 0.5 1 1 1 1)     hex (6 12 13 7 28 34 35 29) (180 7 1) edgeGrading (4 4 4 4 4 1.8 1 1 1.8 1 1 1 1)     hex (7 13 14 8 29 35 36 30) (180 10 1) edgeGrading (4 4 4 4 4 1 1 4 1 1 1 1)     hex (8 14 15 9 30 36 37 31) (180 13 1) simpleGrading (2.5 1 1)     hex (11 17 18 12 33 39 40 34) (25 9 1) simpleGrading (2.5 1 1)     hex (13 19 20 14 35 41 42 36) (25 10 1) simpleGrading (2.5 0.25 1) </pre>
82 83	);
84 85 86	edges ( );
87 88	boundary
89 90 91	( inlet
92 93	type patch; faces
94 95 96 97 98	( (0 22 23 1) (1 23 24 2) (2 24 25 3) );

		*********
( );		
	gePat	chPairs
);		
١.	}	• •
		(14 15 21 20));
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
		(12 13 19 18)
		(10 11 17 16) (11 12 18 17)
		(6 7 13 12) (7 8 14 13) (8 9 15 14)
		$\begin{pmatrix} 6 & 7 & 13 & 12 \\ (7 & 8 & 14 & 12 \end{pmatrix}$
		$(4 \ 5 \ 11 \ 10)$ $(5 \ 6 \ 12 \ 11)$
		$\begin{array}{c} (0 & 1 & 7 & 6) \\ (1 & 2 & 8 & 7) \\ (2 & 3 & 9 & 8) \\ (4 & 5 & 11 & 10) \\ (5 & 6 & 12 & 11) \\ (5 & 6 & 12 & 11) \end{array}$
		$(36 \Lambda 2 \Lambda 3 37)$
		(34 40 41 35) (35 41 42 36)
		(33 39 40 34)
		(30 36 37 31) (32 38 39 33)
		(29 35 36 30)
		(27 33 34 28) (28 34 35 29)
		(24 30 31 25) (26 32 33 27)
		(23 29 30 24)
		( (22 28 29 23)
		faces
	{	type empty;
	fron	tAndBack
	}	);
		(10 16 38 32)
		(5 4 26 27) (4 10 32 26)
		(6 5 27 28)
		( (0 6 28 22)
		faces
	{	type wall;
	lowe	rWall
	}	);
		(15 37 43 21)
		(3 25 31 9) (9 31 37 15)
		type wall; faces
	{	
	} uppe	rWall
	٦	); (10 11 10 12)
		(19 20 42 41) (20 21 43 42)
		(18 19 41 40)
		(16 17 39 38) (17 18 40 39)
		type patch; faces
	{	
	outl	

A major feature of this problem is the use of the full mesh grading capability of blockMesh that is described in section 5.3.1 of the User Guide. The user can see that blocks 4,5 and 6 use the full list of 12 expansion ratios. The expansion ratios correspond to each edge of the block, the first 4 to the edges aligned in the local  $x_1$  direction, the second 4 to the edges in the local  $x_2$  direction and the last 4 to the edges in the local  $x_3$ direction. In blocks 4, 5, and 6, the ratios are equal for all edges in the local  $x_1$  and  $x_3$ 

directions but not for the edges in the  $x_2$  direction that corresponds in all blocks to the global y. If we consider the ratios used in relation to the block definition in section 5.3.1 of the User Guide, we realize that different gradings have been prescribed along the left and right edges in blocks 4,5 and 6 in Figure 3.5. The purpose of this differential grading is to generate a fine mesh close to the most critical region of flow, the corner of the step, and allow it to expand into the rest of the domain.

The mesh can be generated using **blockMesh** from the command line or from within FoamX and viewed as described in previous examples.

#### 3.2.3 Boundary conditions and initial fields

The case files can be viewed, or edited from within FoamX or by hand. In this case, we are required to set the initial and boundary fields for velocity **U**, pressure p, turbulent kinetic energy k and dissipation rate  $\varepsilon$ . The boundary conditions can be specified by setting the physical patch types in FoamX: the upper and lower walls are set to Wall, the left patch to Inlet and the right patch to Outlet. These physical boundary conditions require us to specify a fixedValue at the inlet on **U**, k and  $\varepsilon$ . **U** is given in the problem specification, but the values of k and  $\epsilon$  must be chosen by the user in a similar manner to that described in section 2.1.8.1 of the User Guide. We assume that the inlet turbulence is isotropic and estimate the fluctuations to be 5% of **U** at the inlet. We have

$$U'_x = U'_y = U'_z = \frac{5}{100} 10 = 0.5 \text{ m/s}$$
(3.5)

and

$$k = \frac{3}{2}(0.5)^2 = 0.375 \text{ m}^2/\text{s}^2$$
(3.6)

If we estimate the turbulent length scale l to be 10% of the width of the inlet then

$$\varepsilon = \frac{C_{\mu}^{0.75} k^{1.5}}{l} = \frac{0.09^{0.75} 0.375^{1.5}}{0.1 \times 25.4 \times 10^{-3}} = 14.855 \,\mathrm{m^2/s^3} \tag{3.7}$$

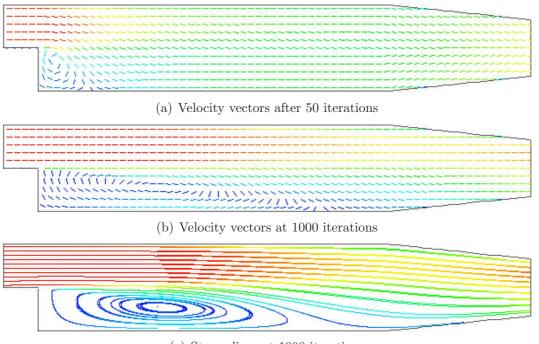
At the outlet we need only specify the pressure p = 0Pa.

#### 3.2.4 Case control

The choices of *fvSchemes* are as follows: the timeScheme should be SteadyState; the gradScheme and laplacianScheme should be set as default to Gauss; and, the divScheme should be set to UD to ensure boundedness.

Special attention should be paid to the settings of *tvTolerances*. Although the top level simpleFoam code contains only equations for p and  $\mathbf{U}$ , the turbulent model solves equations for k,  $\varepsilon$  and  $\mathbf{R}$ , and tolerance settings are required for all 5 equations. A solverTolerance of  $10^{-5}$  and solverRelativeTolerance of 0.1 are acceptable for all variables with the exception of p when  $10^{-6}$  and 0.01 are recommended. Under-relaxation of the solution is required since the problem is steady. A relaxationFactor of 0.7 is acceptable for  $\mathbf{U}$ , k,  $\varepsilon$  and  $\mathbf{R}$  but 0.3 is required for p to avoid numerical instability.

Finally, in *controlDict*, the time step deltaT should be set to 1 since in steady state cases such as this is effectively an iteration counter. With benefit of hindsight we know that the solution requires 1000 iterations reach reasonable convergence, hence endTime is set to 1000. Ensure that the writeFrequency is sufficiently high, *e.g.* 50, that you will not fill the hard disk with data during run time.



(c) Streamlines at 1000 iterations

Figure 3.6: Development of a vortex in the backward-facing step.

#### 3.2.5 Running the case and post-processing

Run the case and post-process the results. After a few iterations, *e.g.* 50, a vortex develops beneath the corner of the step that is the height of the step but narrow in the *x*-direction as shown by the vector plot of velocities is shown Figure 3.6(a). Over several iterations the vortex stretches in the *x*-direction from the step to the outlet until at 1000 iterations the system reaches a steady-state in which the vortex is fully developed as shown in Figure 3.6(b-c).

# 3.3 Supersonic flow over a forward-facing step

In this example we shall investigate supersonic flow over a forward-facing step. The problem description involves a flow of Mach 3 at an inlet to a rectangular geometry with a step near the inlet region that generates shock waves.

This example introduces the following OpenFOAM features for the first time:

• supersonic flow;

# 3.3.1 Problem specification

The problem is defined as follows:

 $Solution \ domain$  The domain is 2 dimensional and consists of a short inlet section followed by a forward-facing step of 20% the height of the section as shown in Figure 3.7

#### Governing equations

• Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.8}$$

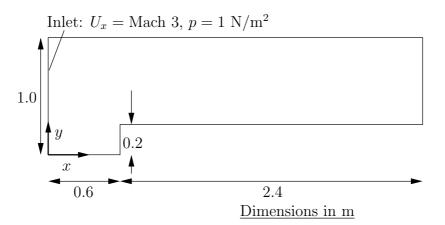


Figure 3.7: Geometry of the forward step geometry

• Ideal gas

$$p = \rho RT \tag{3.9}$$

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \qquad (3.10)$$

• Energy equation for fluid (ignoring some viscous terms),  $e = C_v T$ , with Fourier's Law  $\mathbf{q} = -k\nabla T$ 

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot \left(\rho \mathbf{U} e\right) - \nabla \cdot \left(\frac{k}{C_v}\right) \nabla e = p \nabla \cdot \mathbf{U}$$
(3.11)

Initial conditions U = 0 m/s, p = 1 Pa, T = 1 K.

#### Boundary conditions

- Inlet (left) with fixedValue for velocity U = 3 m/s = Mach 3, pressure p = 1 Pa and temperature T = 1 K;
- Outlet (right) with zeroGradient on U, p and T;
- No-slip adiabatic wall (bottom);
- Symmetry plane (top).

#### Transport properties

• Dynamic viscosity of air  $\mu = 18.1 \mu Pa s$ 

#### Thermodynamic properties

- Specific heat at constant volume  $C_v = 1.78571 \text{ J/kg K}$
- Gas constant R = 0.714286 J/kg K
- Conductivity  $k = 32.3 \ \mu W/m K$

#### Case name forwardStep case located in the \$FOAM\_TUTORIALS/sonicFoam directory.

**Solver name sonicFoam**: an implementation for compressible trans-sonic/supersonic laminar gas flow.

The case is designed such that the speed of sound of the gas  $c = \sqrt{\gamma RT} = 1$  m/s, the consequence being that the velocities are directly equivalent to the Mach number, *e.g.* the inlet velocity of 3 m/s is equivalent to Mach 3. This speed of sound calculation can be verified using the relationship for a perfect gas,  $C_p - Cv = R$ , *i.e.* the ratio of specific heats

$$\gamma = C_p / C_v = \frac{R}{C_v} + 1 \tag{3.12}$$

#### 3.3.2 Mesh generation

The mesh used in this case is relatively simple, specified with uniform rectangular cells of length 0.06 m in the x direction and 0.05 m in the y direction. The geometry can simply be divided into 3 blocks, one below the top of the step, and two above the step, one either side of the step front. The full set of vertices and blocks are given in the mesh description file below:

```
--*- C++ -*----
 1
          _____
\mathbf{2}
                                                      OpenFOAM: The Open Source CFD Toolbox
                          F
                             ield
3
 4
                          0
                             peration
                                                     Version:
                                                                     2.0.0
                           A nd
                                                     Web:
                                                                     www.OpenFOAM.com
                                                   Т
5
                          M anipulation
 6
 7
      FoamFile
 8
9
      ł
                               2.0;
ascii;
10
            version
11
            format
                               dictionarv
            class
12
                               blockMeshDict;
13
            object
14
      11
              *
                    * * *
15
                 *
                                                 *
                                                          *
                                                                                                                            * //
16
      convertToMeters 1;
17
18
19
      vertices
20
             (0 \ 0 \ -0.05)
(0.6 \ 0 \ -0.05)
21
22
             (0 0.2 -0.05)
(0.6 0.2 -0.05)
(3 0.2 -0.05)
(3 0.2 -0.05)
23
24
             (3
(0
25
                    -0.05)
26
                 1
             (0
(3
(0
                .6 1 -0.05)
27
             \begin{array}{c} (0.0 \\ (3 \\ 1 \\ -0.05) \\ (0 \\ 0 \\ 0.05) \\ (0.6 \\ 0 \\ 0.2 \\ 0.05) \\ (0.6 \\ 0.2 \\ 0.05) \\ (3 \\ 0.2 \\ 0.05) \\ (0 \\ 1 \\ 0.05) \\ (0 \\ 1 \\ 0.05) \end{array} 
28
29
30
31
32
33
34
             (ŏ.6
                    1 0.05)
35
                 1 0.05)
36
             (3
      );
37
38
      blocks
39
40
            hex (0 1 3 2 8 9 11 10) (25 10 1) simpleGrading (1 1 1)
41
            hex (2 3 6 5 10 11 14 13) (25 40 1) simpleGrading (1 1 1)
42
            hex (3 4 7 6 11 12 15 14) (100 40 1) simpleGrading (1 1 1)
43
      );
44
45
      edges
46
47
      ();
48
49
      boundary
50
51
      (
             inlet
52
53
             ł
                   type patch;
54
                   faces
55
                   (
56
                          (0 8 10 2)
(2 10 13 5)
57
58
```

```
);
59
        }
60
         outlet
61
62
63
             type patch;
             faces
64
             (
65
                  (4 7 15 12)
66
             );
67
68
         bottom
69
70
             type symmetryPlane;
71
             faces
72
             (
73
                 (0 1 9 8)
74
             );
75
         }
top
76
77
78
         {
79
             type symmetryPlane;
80
             faces
81
                 (5 13 14 6) \\ (6 14 15 7)
82
83
             );
84
85
         }
         obstacle
86
87
88
             type patch;
89
             faces
90
             (
                  (1 \ 3 \ 11 \ 9)
91
                 (3 4 12 11)
92
             );
93
         }
94
     );
95
96
    mergePatchPairs
97
98
    ();
99
100
     11
        101
```

# 3.3.3 Running the case

The case approaches a steady-state at some time after 5 s. The results for pressure at 10 s are shown in Figure 3.8. The results clearly show discontinuities in pressure, *i.e.* shock waves, emanating from ahead of the base of the step.

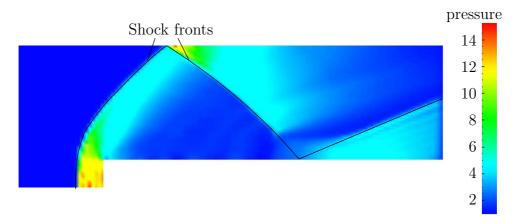


Figure 3.8: Shock fronts in the forward step problem

#### 3.3.4 Exercise

The user can examine the effect on the solution of increasing the inlet velocity.

# 3.4 Decompression of a tank internally pressurised with water

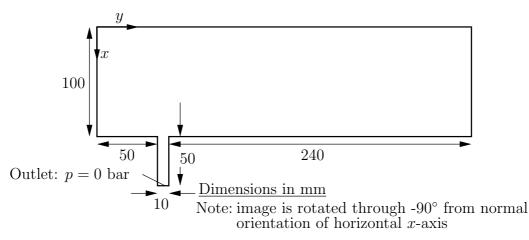
In this example we shall investigate a problem of rapid opening of a pipe valve close to a pressurised liquid-filled tank. The prominent feature of the result in such cases is the propagation of pressure waves which must therefore be modelled as a compressible liquid.

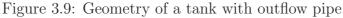
This tutorial introduces the following OpenFOAM features for the first time:

- Mesh refinement
- Pressure waves in liquids

# 3.4.1 Problem specification

 $Solution \ domain$  The domain is 2 dimensional and consists of a tank with a small outflow pipe as shown in Figure 3.9





**Governing equations** This problem requires a model for compressibility  $\psi$  in the fluid in order to be able to resolve waves propagating at a finite speed. A barotropic relationship is used to relate density  $\rho$  and pressure p are related to  $\psi$ .

• Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.13}$$

• The barotropic relationship

$$\frac{\partial \rho}{\partial p} = \frac{\rho}{K} = \psi \tag{3.14}$$

where K is the bulk modulus

• Equation 3.14 is linearised as

$$\rho \approx \rho_0 + \psi \left( p - p_0 \right) \tag{3.15}$$

where  $\rho_0$  and  $p_0$  are the reference density and pressure respectively such that  $\rho(p_0) = \rho_0$ .

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \qquad (3.16)$$

**Boundary conditions** Using FoamX the following physical boundary conditions can be set:

- outerWall is specified the wall condition;
- axis is specified as the symmetryPlane;
- nozzle is specified as a pressureOutlet where p = 0 bar.
- front and back boundaries are specified as empty.

*Initial conditions* U = 0 m/s, p = 100 bar.

#### Transport properties

• Dynamic viscosity of water  $\mu = 1.0$  mPas

#### Thermodynamic properties

- Density of water  $\rho = 1000 \text{ kg/m}^3$
- Reference pressure  $p_0 = 1$  bar
- Compressibility of water  $\psi = 4.54 \times 10^{-7} \text{ s}^2/\text{m}^2$

Solver name sonicLiquidFoam: a compressible sonic laminar liquid flow code.

Case name decompressionTank case located in the \$FOAM\_TUTORIALS/sonicLiquidFoam directory.

#### 3.4.2 Mesh Generation

The full geometry is modelled in this case; the set of vertices and blocks are given in the mesh description file below:

```
----*- C++ -*-
1
\mathbf{2}
        =========
                      F ield
                                            OpenFOAM: The Open Source CFD Toolbox
3
                      O peration
                                          Version: 2.0.0
4
5
                      A nd
                                            Web:
                                                         www.OpenFOAM.com
                      M anipulation
6
            \\/
7
     FoamFile
8
9
     ſ
          version
                          2.0;
10
                          ascii;
          format
11
          class
                          dictionary
12
                          blockMeshDict;
          object
13
     }
14
     1/ *
                * * *
                                                *
                                                  * *
                                                                * * *
                                                                                  * * * * * * * * * //
15
16
     convertToMeters 0.1;
17
18
     vertices
19
20
           (0 \ 0 \ -0.1)
21
           (1 \ 0 \ -0.1)
(0 \ 0.5 \ -0.1)
(1 \ 0.5 \ -0.1)
22
23
24
                0.5 -0.1)
25
              0.6
                   -0.1)
           (0
26
              0.6
                   -0.1)
27
           (1.5
(0 3
(1 3
                0.6 - 0.1)
28
                 -0.1)
29
                 -0.1)
           (1
30
```

```
31
 32
 33
 34
 35
 36
 37
 38
 39
 40
      );
 41
 42
      blocks
 43
 44
      (
           hex (0 1 3 2 10 11 13 12) (30 20 1) simpleGrading (1 1 1)
 45
           hex (2 3 6 5 12 13 16 15) (30 5 1) simpleGrading (1 1 1)
 46
           hex (3 4 7 6 13 14 17 16) (25 5 1) simpleGrading (1 1 1)
 47
 48
           hex (5 6 9 8 15 16 19 18) (30 95 1) simpleGrading (1 1 1)
      );
 49
 50
 51
      edges
      (
);
 52
 53
 54
      boundary
 55
 56
      (
           outerWall
 57
 58
            {
                 type wall;
 59
 60
                 faces
 61
                      62
 63
 64
 65
 66
 67
                );
 68
           }
 69
           axis
{
 70
 71
                 type symmetryPlane;
 72
                 faces
 73
 74
                 (
                      (0 10 12 2)
(2 12 15 5)
(5 15 18 8)
 75
 76
 77
                );
 78
           }
 79
           nozzle
 80
 81
            {
                type patch; faces
 82
 83
                 (
 84
                      (4 7 17 14)
 85
                 );
 86
           }
 87
           back
 88
            {
 89
                 type empty; faces
 90
 91
                 (
 92
                      93
 94
 95
 96
                 );
 97
 98
            }
           front {
99
100
                type empty;
faces
101
102
103
                 (
                      (10 11 13 12)
(12 13 16 15)
(13 14 17 16)
(15 16 19 18)
104
105
106
107
                 );
108
           }
109
110
      );
111
      mergePatchPairs
112
      (
);
113
114
115
```

In order to improve the numerical accuracy, we shall use the reference level of 1 bar for the pressure field. Note that both the internal field level and the boundary conditions are offset by the reference level.

#### 3.4.3 Preparing the Run

Before we commence the setup of the calculation, we need to consider the characteristic velocity of the phenomenon we are trying to capture. In the case under consideration, the fluid velocity will be very small, but the pressure wave will propagate with the speed of sound in water. The speed of sound is calculated as:

$$c = \sqrt{\frac{1}{\psi}} = \sqrt{\frac{1}{4.54 \times 10^{-7}}} = 1483.2$$
m/s. (3.17)

For the mesh described above, the characteristic mesh size is approximately 2 mm (note the scaling factor of 0.1 in the *blockMeshDict* file). Using

$$Co = \frac{U\,\Delta t}{\Delta x} \tag{3.18}$$

a reasonable time step is around  $\Delta t = 5 \times 10^{-7}$ s, giving the *Co* number of 0.35, based on the speed of sound. Also, note that the reported *Co* number by the code (associated with the convective velocity) will be two orders of magnitude smaller. As we are interested in the pressure wave propagation, we shall set the simulation time to 0.25 ms. For reference, the *controlDict* file is quoted below.

```
----*- C++ -*-----
 1
 2
3
                   F
                      ield
                                        OpenFOAM: The Open Source CFD Toolbox
                   0 peration
                                        Version:
                                                   2.0.0
 4
                    A nd
                                        Web:
                                                   www.OpenFOAM.com
5
                   M anipulation
 6
                                     7
    FoamFile
 8
9
     ł
                       2.0;
         version
10
                       ascii;
11
         format
                       dictionary;
         class
12
         location
                        "system"
13
                       controlDict:
         object
14
15
    }
// *
                                                                                          * * //
          *
            *
               *
                 *
                                                       *
                                                         *
                                                           *
                                                              *
16
17
     application
                       sonicLiquidFoam;
18
19
     startFrom
                       startTime;
20
^{21}
     startTime
                       0;
22
23
     stopAt
                       endTime;
24
25
     endTime
                       0.0001;
26
27
     deltaT
                       5e-07;
28
29
    writeControl
                       timeStep;
30
31
    writeInterval
                       20;
32
33
                       0;
    purgeWrite
34
35
    writeFormat
                       ascii;
36
37
    writePrecision
                       6;
38
39
     writeCompression off;
40
41
```

# 3.4.4 Running the case

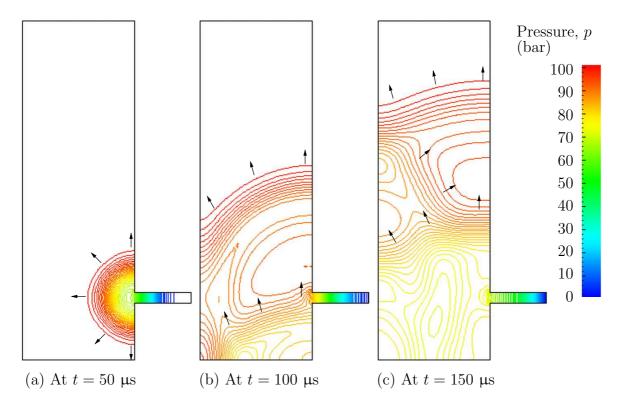


Figure 3.10: Propagation of pressure waves

The user can run the case and view results in dxFoam. The liquid flows out through the nozzle causing a wave to move along the nozzle. As it reaches the inlet to the tank, some of the wave is transmitted into the tank and some of it is reflected. While a wave is reflected up and down the inlet pipe, the waves transmitted into the tank expand and propagate through the tank. In Figure 3.10, the pressures are shown as contours so that the wave fronts are more clearly defined than if plotted as a normal isoline plot.

If the simulation is run for a long enough time for the reflected wave to return to the pipe, we can see that negative absolute pressure is detected. The modelling permits this and has some physical basis since liquids can support tension, *i.e.* negative pressures. In reality, however, impurities or dissolved gases in liquids act as sites for cavitation, or vapourisation/boiling, of the liquid due to the low pressure. Therefore in practical situations, we generally do not observe pressures falling below the vapourisation pressure of the liquid; not at least for longer than it takes for the cavitation process to occur.

# 3.4.5 Improving the solution by refining the mesh

Looking at the evolution of the resulting pressure field in time, we can clearly see the propagation of the pressure wave into the tank and numerous reflections from the inside walls. It is also obvious that the pressure wave is smeared over a number of cells. We shall

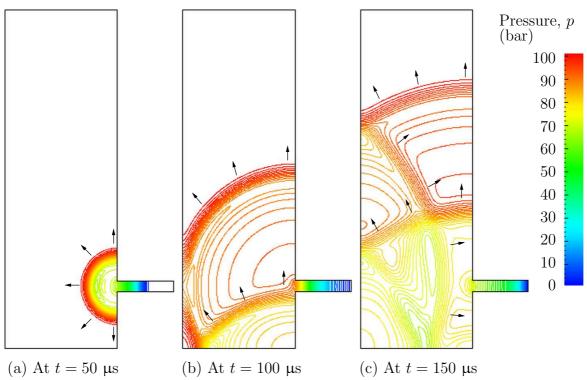


Figure 3.11: Propagation of pressure waves with refined mesh

now refine the mesh and reduce the time step to obtain a sharper front resolution. Simply edit the *blockMeshDict* and increase the number of cells by a factor of 4 in the x and y directions, *i.e.* block 0 becomes (120 80 1) from (30 20 1) and so on. Run blockMesh on this file. In addition, in order to maintain a Courant number below 1, the time step must be reduced accordingly to  $\Delta t = 10^{-7}$  s. The second simulation gives considerably better resolution of the pressure waves as shown in Figure 3.11.

# 3.5 Magnetohydrodynamic flow of a liquid

In this example we shall investigate an flow of an electrically-conducting liquid through a magnetic field. The problem is one belonging to the branch of fluid dynamics known as magnetohydrodynamics (MHD) that uses mhdFoam.

#### 3.5.1 Problem specification

The problem is known as the Hartmann problem, chosen as it contains an analytical solution with which mhdFoam can be validated. It is defined as follows:

**Solution domain** The domain is 2 dimensional and consists of flow along two parallel plates as shown in Fig. 3.12.

#### Governing equations

• Mass continuity for incompressible fluid

$$\nabla \cdot \mathbf{U} = 0 \tag{3.19}$$

• Momentum equation for incompressible fluid

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{U}) + \nabla \cdot (2\mathbf{B}\Gamma_{\mathbf{B}\mathbf{U}}\mathbf{B}) + \nabla \cdot (\nu\mathbf{U}) + \nabla (\Gamma_{\mathbf{B}\mathbf{U}}\mathbf{B}\mathbf{:}\mathbf{B}) = -\nabla p \quad (3.20)$$

P-67

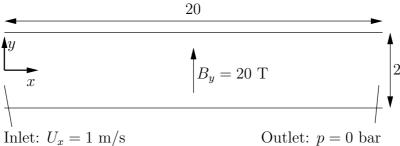


Figure 3.12: Geometry of the Hartmann problem

where **B** is the magnetic flux density,  $\Gamma_{\mathbf{BU}} = (2\mu\rho)^{-1}$ .

• Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3.21}$$

where  $\mathbf{E}$  is the electric field strength.

$$\nabla \cdot \mathbf{B} = 0 \tag{3.22}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$$
(3.23)

assuming  $\partial \mathbf{D} / \partial t \ll \mathbf{J}$ . Here, **H** is the magnetic field strength, **J** is the current density and **D** is the electric flux density.

• Charge continuity

$$\nabla \cdot \mathbf{J} = 0 \tag{3.24}$$

• Constitutive law

$$\mathbf{B} = \mu \mathbf{H} \tag{3.25}$$

• Ohm's law

$$\mathbf{J} = \sigma \left( \mathbf{E} + \mathbf{U} \times \mathbf{B} \right) \tag{3.26}$$

• Combining Equation 3.21, Equation 3.23, Equation 3.26, and taking the curl

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}) - \nabla \cdot (\phi_{\mathbf{B}}\mathbf{U}) - \nabla \cdot (\Gamma_{\mathbf{B}}\mathbf{B}) = 0$$
(3.27)

#### **Boundary** conditions

- inlet is specified the inlet condition with fixed velocity  $\mathbf{U} = (1, 0, 0)$  m/s;
- outlet is specified as the outlet with with fixed pressure p = 0 Pa;
- upperWall is specified as a wall where  $\mathbf{B} = (0, 20, 0)$  T.
- lowerWall is specified as a wall where  $\mathbf{B} = (0, 20, 0)$  T.
- front and back boundaries are specified as empty.

*Initial conditions* U = 0 m/s, p = 100 Pa, B = (0, 20, 0) T.

#### Transport properties

- Kinematic viscosity  $\nu = 1$  Pas
- Density  $\rho = 1 \text{ kg m/s}$
- Electrical conductivity  $\sigma = 1 \ (\Omega \,\mathrm{m})^{-1}$
- Permeability  $\mu = 1$  H/m

Solver name mhdFoam: an incompressible laminar magneto-hydrodynamics code.

Case name hartmann case located in the \$FOAM\_TUTORIALS/mhdFoam directory.

#### 3.5.2 Mesh generation

The geometry is simply modelled with 100 cells in the x-direction and 40 cells in the y-direction; the set of vertices and blocks are given in the mesh description file below:

```
----*- C++ -*----
 1
2
                      F
                                            OpenFOAM: The Open Source CFD Toolbox
                        ield
        //
3
 4
                      O peration
                                            Version:
                                                         2.0.0
                                                         www.OpenFOAM.com
                      A nd
 \mathbf{5}
                                         Web:
                      M anipulation
     \langle \rangle
                                         6
                                               _____
 7
                                                                           _____
 8
     FoamFile
9
     ł
                          2.0;
10
          version
11
          format
                          ascii;
                          dictionary;
blockMeshDict;
12
          class
13
          object
14
     11
                                                                                                * * * //
                                                *
15
16
     convertToMeters 1;
17
18
     vertices
19
20
     (
           (0 -1 0)
(20 -1 0)
21
22
           (20 \ 1 \ 0)
23
           (0 \ 1 \ 0)
(0 \ -1 \ 0.1)
^{24}
25
           (20
(20
               -1 \ 0.1)
1 0.1)
26
27
           (0)
             1 0.1)
28
     );
29
30
     blocks
31
32
          hex (0 1 2 3 4 5 6 7) (100 40 1) simpleGrading (1 1 1)
33
     );
34
35
     edges
36
37
     );
38
39
     boundary
40
41
     (
42
          inlet
43
          {
               type patch;
44
45
               faces
46
                (
                     (0 4 7 3)
47
               );
48
          }
49
          outlet
50
51
          ł
               type patch;
52
               faces
53
54
                (
                     (2 6 5 1)
55
               );
56
57
          ĺowerWall
{
58
59
```

60	type patch;
61	faces
62	
63	(1 5 4 0)
64	);
65	}
66	upperWall
67	{
68	type patch;
69	faces
70	(
71	(3 7 6 2)
72	);
73	}
74	frontAndBack
75	{
76	type empty;
77	faces
78	
79	$ \begin{pmatrix} 0 & 3 & 2 & 1 \\ (4 & 5 & 6 & 7) \end{pmatrix} $
80	
81	);
82	}
83	);
84	
85	mergePatchPairs
86	(
87	);
88	//
89	// ************************************

#### 3.5.3 Running the case

The user can run the case and view results in dxFoam. It is also useful at this stage to run the Ucomponents utility to convert the U vector field into individual scalar components. MHD flow is governed by, amongst other things, the Hartmann number which is a measure of the ratio of electromagnetic body force to viscous force

$$M = BL \sqrt{\frac{\sigma}{\rho\nu}} \tag{3.28}$$

where L is the characteristic length scale. In this case with  $B_y = 20$  T, M = 20 and the electromagnetic body forces dominate the viscous forces. Consequently with the flow fairly steady at t = 2 s the velocity profile is almost planar, viewed at a cross section midway along the domain x = 10 m. The user can plot a graph of the profile of  $U_x$  in dxFoam. Now the user should reduce the magnetic flux density **B** to 1 Tand re-run the code and **Ucomponents**. In this case, M = 1 and the electromagnetic body forces no longer dominate. The velocity profile consequently takes on the parabolic form, characteristic of Poiseuille flow as shown in Figure 3.13. To validate the code the analytical solution for the velocity profile  $U_x$  is superimposed in Figure 3.13, given by:

$$\frac{U_x(y)}{U_x(0)} = \frac{\cosh M - \cosh M(y/L)}{\cosh M - 1}$$
(3.29)

where the characteristic length L is half the width of the domain, *i.e.* 1 m.

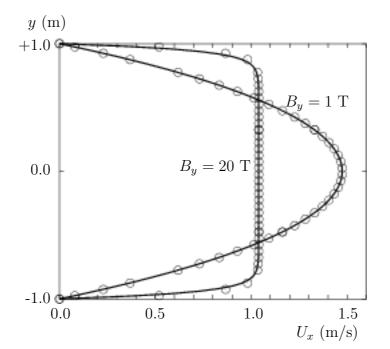


Figure 3.13: Velocity profile in the Hartmann problem for  $B_y = 1$  T and  $B_y = 20$  T.

 $\mathsf{Open}\nabla\mathsf{FOAM}\text{-}2.0.0$ 

# Index

\*

#### Symbols Numbers A B C D E F G H I J K L M N O P Q R S T U V W X Z

#### Symbols

tensor member function, P-25 + tensor member function, P-25 tensor member function, P-25 / tensor member function, P-25 /\*...\*/ C++ syntax, U-78 11 C++ syntax, U-78 OpenFOAM file syntax, U-104 # include C++ syntax, U-72, U-78 & tensor member function, P-25 && tensor member function, P-25 tensor member function, P-25 <LESModel>Coeffs keyword, U-184 <RASModel>Coeffs keyword, U-183 <delta>Coeffs keyword, U-184 0.000000e+00 directory, U-104 1-dimensional mesh, U-130 1D mesh, U-130 2-dimensional mesh, U-130 2D mesh, U-130

#### Numbers

 $\theta$  directory, U-104

## Α

access functions, P-23 addLayersControls keyword, U-146 adiabaticFlameT utility, U-96 adjointShapeOptimizationFoam solver, U-85 adjustableRunTime keyword entry, U-62, U-111 adjustTimeStep keyword, U-62 agglomerator keyword, U-122 algorithms tools, U-96 alphaContactAngle boundary condition, U-59 analytical solution, P-45 Animations window panel, U-168 anisotropicFilter model, U-101 Annotation window panel, U-26, U-167 ansysToFoam utility, U-90 APIfunctions model, U-100 applications, U-69 Apply button, U-164, U-168 applyBoundaryLayer utility, U-90 applyWallFunctionBoundaryConditions utility, U-90 arbitrarily unstructured, P-31 arc keyword entry, U-139 arc keyword, U-138 As keyword, U-182 ascii keyword entry, U-112 attachMesh utility, U-91 Auto Accept button, U-168 autoMesh library, U-97 autoPatch utility, U-91 autoRefineMesh utility, U-92 axes right-handed, U-136 right-handed rectangular Cartesian, P-15, U-20 axi-symmetric cases, U-135, U-144 axi-symmetric mesh, U-130

## В

background process, U-26, U-81 backward keyword entry, U-119 Backward differencing, P-39 barotropicCompressibilityModels library, U-99 basicMultiComponentMixture U-99. model, U-180 basicSolidThermo library, U-100 basicThermophysicalModels library, U-98 binary keyword entry, U-112 BirdCarreau model, U-102 blended differencing, P-38 block expansion ratio, U-140 block keyword, U-138 blocking keyword entry, U-80 blockMesh library, U-97 blockMesh solver, P-47 blockMesh utility, U-39, U-90, U-136 blockMesh executable vertex numbering, U-140 blockMeshDict dictionary, U-20, U-22, U-37, U-50, U-136, boxToCell keyword, U-60 U-144 blocks keyword, U-22, U-32, U-140 boundaries, U-132 boundary, U-132 boundary dictionary, U-129, U-136 boundary keyword, U-141 boundary condition alphaContactAngle, U-59 calculated, U-136 cyclic, U-135, U-142 directionMixed, U-136 empty, P-63, P-68, U-20, U-130, U-135 fixedGradient, U-136 fixedValue, U-136 fluxCorrectedVelocity, U-137 inlet, P-68 inletOutlet. U-137 mixed, U-136 movingWallVelocity, U-137 outlet, P-68 outletInlet, U-137 partialSlip, U-137 patch, U-135 pressureDirectedInletVelocity, U-137 pressureInletVelocity, U-137 pressureOutlet, P-63 pressureTransmissive, U-137 processor, U-135 setup, U-22 slip, U-137

supersonicFreeStream, U-137 surfaceNormalFixedValue, U-137 symmetryPlane, P-63, U-135 totalPressure, U-137 turbulentInlet, U-137 wall, U-42 wall, P-63, P-68, U-59, U-135 wallBuoyantPressure, U-137 wedge, U-130, U-135, U-144 zeroGradient, U-136 boundary conditions, P-43 Dirichlet, P-43 inlet, P-44 Neumann, P-43 no-slip impermeable wall, P-44 outlet, P-44 physical, P-44 symmetry plane, P-44 boundaryField keyword, U-23, U-108 boundaryFoam solver, U-85 bounded keyword entry, U-117, U-118 boxTurb utility, U-90 breaking of a dam, U-57 bubbleFoam solver, U-87 buoyantBaffleSimpleFoam solver, U-88 buoyantBoussinesqPimpleFoam solver, U-88 buoyantBoussinesqSimpleFoam solver, U-88 buoyantPimpleFoam solver, U-88 buoyantSimpleFoam solver, U-88 buoyantSimpleRadiationFoam solver, U-88 button Apply, U-164, U-168 Auto Accept, U-168 Choose Preset, U-166 Delete, U-164 Edit Color Map, U-165 Enable Line Series, U-36 Orientation Axes, U-26, U-167 Refresh Times, U-27 Rescale to Data Range, U-27 Reset, U-164 Set Ambient Color, U-166 Update GUI, U-165 Use Parallel Projection, U-26 Use parallel projection, U-167

#### C

C++ syntax /\*...\*/, U-78 //, U-78 # include, U-72, U-78 cacheAgglomeration keyword, U-123 calculated boundary condition, U-136 cAlpha keyword, U-63 cases, U-103 castellatedMesh keyword, U-146 castellatedMeshControls dictionary, U-147-U-149 castellatedMeshControls keyword, U-146 cavitatingFoam solver, U-87 cavity flow, U-19 **CEI\_ARCH** environment variable, U-173 CEI\_HOME environment variable, U-173 cell expansion ratio, U-140 cell class, P-31 cell keyword entry, U-174 cellLimited keyword entry, U-117 cellPoint keyword entry, U-174 cellPointFace keyword entry, U-174 cells dictionary, U-136 central differencing, P-38 cfdTools tools, U-97 cfx4ToFoam utility, U-90, U-154 changeDictionary utility, U-90 channelFoam solver, U-85 Charts window panel, U-168 checkMesh utility, U-91, U-155 chemFoam solver, U-88 chemistryModel library, U-100 chemistryModel model, U-100 chemistrySolver model, U-100 chemkinToFoam utility, U-96 Choose Preset button, U-166 chtMultiRegionFoam solver, U-88 Chung library, U-99 class cell, P-31 dimensionSet, P-25, P-32, P-33 face, P-31 finiteVolumeCalculus, P-33 finiteVolumeMethod, P-33 fvMesh, P-31 fvSchemes, P-36 fvc, P-36 fvm, P-36

pointField, P-31 polyBoundaryMesh, P-31 polyMesh, P-31, U-127, U-129 polyPatchList, P-31 polyPatch, P-31 scalarField, P-29 scalar, P-23 slice, P-31 symmTensorField, P-29 symmTensorThirdField, P-29 tensorField, P-29 tensorThirdField, P-29 tensor, P-23 vectorField, P-29 vector, P-23, U-107 word, P-25, P-31 class keyword, U-105 clockTime keyword entry, U-111 cloud keyword, U-175 cmptAv tensor member function, P-25 Co utility, U-93 coalChemistryFoam solver, U-88 coalCombustion library, U-98 cofactors tensor member function, P-25 coldEngineFoam solver, U-88 collapseEdges utility, U-92 Color By menu, U-166 Color Legend window, U-29 Color Legend window panel, U-166 Color Scale window panel, U-166 Colors window panel, U-168 combinePatchFaces utility, U-92 comments, U-78 commsType keyword, U-80 compressed keyword entry, U-112 compressibleInterFoam solver, U-87 compressibleLESModels library, U-102 compressibleRASModels library, U-101 constant directory, U-104, U-179 constLaminarFlameSpeed model, U-99 constTransport model, U-100, U-180 containers tools, U-96 continuum mechanics, P-15 control of time, U-111 controlDict

dictionary, P-65, U-23, U-32, U-43, U-52, ddt U-62, U-104, U-160 controlDict file, P-50 convection, see divergence, P-38 convergence, U-40 conversion library, U-98 convertToMeters keyword, U-138 coordinate system, P-15 coordinate system, U-20 corrected keyword entry, U-117, U-118 Courant number, P-42, U-24 Cp keyword, U-181 cpuTime keyword entry, U-111 Crank Nicholson temporal discretisation, P-42 CrankNicholson keyword entry, U-119 createBaffles utility, U-91 createPatch utility, U-91 createTurbulenceFields utility, U-93 cross product, *see* tensor, vector cross product CrossPowerLaw keyword entry, U-61 CrossPowerLaw model, U-102 cubeRootVolDelta model, U-101 cubicCorrected keyword entry, U-119 cubicCorrection keyword entry, U-116 curl, P-37 curl fvc member function, P-37 Current Time Controls menu, U-27, U-165 curve keyword, U-175 Cv keyword, U-181 cyclic boundary condition, U-135, U-142 cyclic keyword entry, U-134 cvlinder flow around a, P-45

## D

d2dt2 fvc member function, P-37 fvm member function, P-37 dam breaking of a, U-57 datToFoam utility, U-90 db tools, U-96

fvc member function, P-37 fvm member function, P-37 DeardorffDiffStress model, U-102 debug keyword, U-146 decomposePar utility, U-81, U-82, U-95 decomposeParDict dictionary, U-81 decomposition of field, U-81 of mesh, U-81 decompositionMethods library, U-98 decompression of a tank, P-62 defaultFieldValues keyword, U-60 deformedGeom utility, U-91 Delete button, U-164 delta keyword, U-83, U-184 deltaT keyword, U-111 dependencies, U-72 dependency lists, U-72 det tensor member function, P-25 determinant, see tensor, determinant dev tensor member function, P-25 diag tensor member function, P-25 diagonal keyword entry, U-121, U-122 DIC keyword entry, U-122 DICGaussSeidel keyword entry, U-122 dictionary LESProperties, U-183 PISO, U-25 blockMeshDict, U-20, U-22, U-37, U-50, U-136, U-144 boundary, U-129, U-136 castellatedMeshControls, U-147–U-149 cells, U-136 controlDict, P-65, U-23, U-32, U-43, U-52, U-62, U-104, U-160 decomposeParDict, U-81 faces, U-129, U-136 fvSchemes, U-62, U-63, U-104, U-113 fvSolution, U-104, U-120 mechanicalProperties, U-51 neighbour, U-129 owner, U-129 points, U-129, U-136 thermalProperties, U-52 thermophysicalProperties, U-179

transportProperties, U-23, U-40, U-43 turbulenceProperties, U-42, U-61, U-183 dieselEngineFoam solver, U-88 dieselFoam solver, U-88 dieselMixture model, U-99, U-180 dieselSpray library, U-98 differencing Backward, P-39 blended, P-38 central, P-38 Euler implicit, P-39 Gamma, P-38 MINMOD, P-38 SUPERBEE, P-38 upwind, P-38 van Leer, P-38 DILU keyword entry, U-122 dimension checking in OpenFOAM, P-25, U-107 dimensional units, U-107 dimensioned < Type > template class, P-25 dimensionedTypes tools, U-97 dimensions keyword, U-23, U-108 dimensionSet class, P-25, P-32, P-33 dimensionSet tools, U-97 directionMixed boundary condition, U-136 directory 0.000000e+00, U-104 0, U-104 Make, U-73 constant, U-104, U-179 fluentInterface, U-170 polyMesh, U-104, U-129 processorN, U-82 run, U-103 system, P-50, U-104 tutorials, P-45, U-19 discretisation equation, P-33 panel, U-25, Display window U-164, U-165 distance keyword entry, U-149, U-175 distributed model, U-98 distributed keyword, U-83, U-84 distributionModels library, U-98 div fvc member function, P-37 fvm member function, P-37 divergence, P-37, P-39

divSchemes keyword, U-114 dnsFoam solver, U-87 doLayers keyword, U-146 double inner product, see tensor.double inner product dsmc library, U-98 dsmcFieldsCalc utility, U-94 dsmcFoam solver, U-89 dsmcInitialise utility, U-90 dx keyword entry, U-174 dynamicFvMesh library, U-97 dynamicMesh library, U-97 dynLagrangian model, U-101 dynMixedSmagorinsky model, U-101 dynOneEqEddy model, U-101, U-102 dynSmagorinsky model, U-101

#### $\mathbf{E}$

eConstThermo model, U-100, U-179 edgeGrading keyword, U-140 edgeMesh library, U-97 edges keyword, U-138 Edit menu, U-167, U-168 Edit Color Map button, U-165 egrMixture model, U-99, U-180 electrostaticFoam solver, U-89 empty boundary condition, P-63, P-68, U-20, U-130, U-135 empty keyword entry, U-134 Enable Line Series button, U-36 endTime keyword, U-24, U-111 engine library, U-98 engineCompRatio utility, U-94 engineFoam solver, U-88 U-27, engineSwirl utility, U-90 ensight74FoamExec utility, U-172 ENSIGHT7\_INPUT environment variable, U-173 ENSIGHT7\_READER environment variable, U-173 ensightFoamReader utility, U-92 enstrophy utility, U-93 environment variable CEI\_ARCH, U-173 CEI\_HOME, U-173

ENSIGHT7\_INPUT, U-173

ENSIGHT7\_READER, U-173 FOAM\_RUN, U-103 WM\_ARCH\_OPTION, U-76 WM\_ARCH, U-76 WM\_COMPILER\_BIN, U-76 WM\_COMPILER\_DIR, U-76 WM\_COMPILER\_LIB, U-76 WM\_COMPILER, U-76 WM\_COMPILE\_OPTION, U-76 WM\_DIR, **U-76** WM\_MPLIB, U-76 WM\_OPTIONS, U-76 WM\_PRECISION\_OPTION, U-76 WM\_PROJECT\_DIR, U-76 WM\_PROJECT\_INST\_DIR, U-76 WM\_PROJECT\_USER\_DIR, U-76 WM\_PROJECT\_VERSION, U-76 WM\_PROJECT, U-76 wmake, U-75 ePsiThermo model, U-98, U-180 equilibriumCO utility, U-96 equilibriumFlameT utility, U-96 errorReduction keyword, U-153 Euler keyword entry, U-119 Euler implicit differencing, P-39 temporal discretisation, P-42 examples decompression of a tank, P-62 flow around a cylinder, P-45 flow over backward step, P-51 Hartmann problem, P-67 supersonic flow over forward step, P-58 execFlowFunctionObjects utility, U-94 expandDictionary utility, U-96 expansionRatio keyword, U-152 explicit temporal discretisation, P-42 extrude2DMesh utility, U-90 extrudeMesh utility, U-90

## $\mathbf{F}$

extrudeToRegionMesh utility, U-90

face class, P-31 face keyword, U-175 faceAgglomerate utility, U-90 faceAreaPair keyword entry, U-122 faceLimited keyword entry, U-117 faces dictionary, U-129, U-136 FDIC

keyword entry, U-122 featureAngle keyword, U-152 features keyword, U-147, U-148 field U, U-24 p, U-24 decomposition, U-81 FieldField<Type> template class, P-32 fieldFunctionObjects library, U-97 fields, P-29 mapping, U-160 fields tools, U-97 fields keyword, U-174 Field<Type> template class, P-29 fieldValues keyword, U-60 fieldview9Reader utility, U-92 file Make/files, U-74 controlDict, P-50 files, U-73 g, U-61 options, U-73 snappyHexMeshDict, U-145 transportProperties, U-61 file format, U-104 fileFormats library, U-98 fileModificationChecking keyword, U-80 fileModificationSkew keyword, U-80 files file, U-73 filteredLinear2 keyword entry, U-116 finalLayerRatio keyword, U-152 financialFoam solver, U-89 finite volume discretisation, P-27 mesh, P-31finiteVolume library, U-97 finiteVolume tools. U-97 finiteVolumeCalculus class, P-33 finiteVolumeMethod class, P-33 fireFoam solver, U-88 firstTime keyword, U-111 fixed keyword entry, U-112 fixedGradient boundary condition, U-136 fixedValue boundary condition, U-136 flattenMesh utility, U-91 floatTransfer keyword, U-80 flow

free surface, U-57 laminar, U-19 steady, turbulent, P-53 supersonic, P-58 turbulent, U-19 flow around a cylinder, P-45 flow over backward step, P-51 flowType utility, U-93 fluent3DMeshToFoam utility, U-90 fluentInterface directory, U-170 fluentMeshToFoam utility, U-90, U-154 fluxCorrectedVelocity boundary condition, U-137 fluxRequired keyword, U-114 **OpenFOAM** cases, U-103 FOAM\_RUN environment variable, U-103 foamCalc utility, U-35, U-94 foamCalcFunctions library, U-97 foamCorrectVrt script/alias, U-158 foamDataToFluent utility, U-92, U-170 foamDebugSwitches utility, U-96 FoamFile keyword, U-105 foamFile keyword entry, U-174 foamFormatConvert utility, U-96 foamInfoExec utility, U-96 foamJob script/alias, U-177 foamListTimes utility, U-94 foamLog script/alias, U-177 foamMeshToFluent utility, U-90, U-170 foamToEnsight utility, U-92 foamToEnsightParts utility, U-92 foamToFieldview9 utility, U-92 foamToGMV utility, U-92 foamToStarMesh utility, U-90 foamToSurface utility, U-91 foamToTecplot360 utility, U-92 foamToVTK utility, U-92 foamUpgradeCyclics utility, U-90 foamUpgradeFvSolution utility, U-90 forces library, U-97 foreground process, U-26 format keyword, U-105 fourth keyword entry, U-117, U-118 functions keyword, U-112 fvc class, P-36 fvc member function curl, P-37

d2dt2, P-37 ddt, P-37div, P-37 gGrad, P-37 grad, P-37 laplacian, P-37 lsGrad, P-37 snGrad, P-37 snGradCorrection, P-37 sqrGradGrad, P-37 fvDOM library, U-99 fvm class, P-36 fvm member function d2dt2, P-37 ddt. P-37 div, P-37 laplacian, P-37 Su, P-37 SuSp, P-37 fvMatrices tools, U-97 fvMatrix template class, P-33 fvMesh class, P-31 fvMesh tools. U-97 **fvMotionSolvers** library, U-97 fvSchemes dictionary, U-62, U-63, U-104, U-113 fvSchemes class, P-36 fvSchemes menu entry, U-53 fvSolution dictionary, U-104, U-120

## G

g file, U-61 gambitToFoam utility, U-91, U-154 GAMG keyword entry, U-54, U-121, U-122 Gamma keyword entry, U-116 Gamma differencing, P-38 Gauss keyword entry, U-117 Gauss's theorem, P-36 GaussSeidel keyword entry, U-122 General window panel, U-167, U-168 general keyword entry, U-112 genericFvPatchField library, U-98 geometric-algebraic multi-grid, U-122 GeometricBoundaryField template class, P-32 geometricField<Type> template class, P-32 geometry keyword, U-146 gGrad fvc member function, P-37 global tools, U-97 gmshToFoam utility, U-91 gnuplot keyword entry, U-112, U-174 grad fvc member function, P-37 (Grad Grad) squared, P-37 gradient, P-37, P-40 Gauss scheme, P-40 Gauss's theorem, U-53 least square fit, U-53 least squares method, P-40, U-53 surface normal, P-40 gradSchemes keyword, U-114 graph tools, U-97 graphFormat keyword, U-112 GuldersEGRLaminarFlameSpeed model, U-99 GuldersLaminarFlameSpeed model, U-99

## Η

hConstThermo model, U-100, U-179 Help menu, U-167 HerschelBulkley model, U-102 Hf keyword, U-181 hhuMixtureThermo model, U-99, U-180 hierarchical keyword entry, U-82, U-83 highCpCoeffs keyword, U-182 homogenousDynSmagorinsky model, U-101 homogeneousMixture model, U-99, U-180 hPolynomialThermo model, U-100, U-179 hPsiMixtureThermo model, U-99, U-180 hPsiThermo model, U-98, U-180 hRhoMixtureThermo model, U-99, U-180 hRhoThermo model, U-98, U-180 hsPsiMixtureThermo model, U-99, U-180 hsPsiThermo model, U-98, U-180 hsRhoMixtureThermo model, U-99, U-180

# Ι

Ι

tensor member function, P-25 icoFoam solver, U-19, U-23, U-24, U-26, U-85 icoPolynomial model, U-100, U-179 icoUncoupledKinematicParcelDyMFoam solver, **U-88** icoUncoupledKinematicParcelFoam solver, U-88 kEpsilon model, U-100, U-101 ideasToFoam utility, U-154 ideasUnvToFoam utility, U-91 identities, see tensor, identities

identity, see tensor, identity incompressibleLESModels library, U-101 incompressibleRASModels library, U-100 incompressibleTransportModels library, P-54, U-102 incompressibleTurbulenceModels library, P-54 index notation, P-16, P-17 Information window panel, U-164 inhomogeneousMixture model, U-99, U-180 inlet boundary condition, P-68 inletOutlet boundary condition, U-137 inner product, see tensor, inner product inotify keyword entry, U-80 inotifyMaster keyword entry, U-80 inside keyword entry, U-149 insideCells utility, U-91 interDyMFoam solver, U-87 interfaceProperties library, U-102 interfaceProperties model, U-102 interFoam solver, U-87 interMixingFoam solver, U-87 internalField keyword, U-23, U-108 interPhaseChangeFoam solver, U-87 interpolation tools, U-97 interpolationScheme keyword, U-174 interpolations tools, U-97 interpolationSchemes keyword, U-114 inv tensor member function, P-25 iterations maximum, U-121

## J

janafThermo model, U-100, U-179 jobControl library, U-97 jplot keyword entry, U-112, U-174

#### $\mathbf{K}$

keyword As, U-182 Cp, U-181 Cv, U-181 FoamFile, U-105 Hf, U-181 LESModel, U-184 Pr, U-182 RASModel, U-183 Tcommon, U-182 Thigh, U-182 Tlow, U-182 Ts, U-182 addLayersControls, U-146 adjustTimeStep, U-62agglomerator, U-122 arc, U-138 blocks, U-22, U-32, U-140 block, U-138 boundaryField, U-23, U-108 boundary, U-141 boxToCell, U-60 cAlpha, U-63 cacheAgglomeration, U-123 castellatedMeshControls, U-146 castellatedMesh, U-146 class, U-105 cloud, U-175 commsType, U-80 convertToMeters, U-138 curve, U-175 debug, U-146 defaultFieldValues, U-60 deltaT, U-111 delta, U-83, U-184 dimensions, U-23, U-108 distributed, U-83, U-84 divSchemes, U-114 doLayers, U-146 edgeGrading, U-140 edges, U-138 endTime, U-24, U-111 errorReduction, U-153 expansionRatio, U-152 face, U-175 featureAngle, U-152 features, U-147, U-148 fieldValues, U-60 fields, U-174 fileModificationChecking, U-80 fileModificationSkew, U-80 finalLayerRatio, U-152 firstTime, U-111 floatTransfer, U-80 fluxRequired, U-114 format, U-105 functions, U-112

geometry, U-146 gradSchemes, U-114 graphFormat, U-112 highCpCoeffs, U-182 internalField, U-23, U-108 interpolationSchemes, U-114 interpolationScheme, U-174 laplacianSchemes, U-114 latestTime, U-40 layers, U-152 leastSquares, U-53 levels, U-150 libs, U-80, U-112 locationInMesh, U-148, U-149 location, U-105 lowCpCoeffs, U-182 manualCoeffs, U-83 maxAlphaCo, U-62 maxBoundarySkewness, U-153 maxConcave, U-153 maxCo, U-62 maxDeltaT, U-62 maxFaceThicknessRatio, U-152 maxGlobalCells, U-148 maxInternalSkewness, U-153 maxIter, U-121 maxLocalCells, U-148 maxNonOrtho, U-153 maxThicknessToMedialRatio, U-152 mergeLevels, U-123 mergePatchPairs, U-138 mergeTolerance, U-146 meshQualityControls, U-146 method, U-83midPointAndFace, U-175 midPoint, U-175 minArea, U-153 minDeterminant, U-153 minFaceWeight, U-153 minFlatness, U-153 minMedianAxisAngle, U-152 minRefinementCells, U-148 minThickness, U-152 minTriangleTwist, U-153 minTwist, U-153 minVolRatio, U-153 minVol, U-153mode, U-149molWeight, U-181 mu, U-182 nAlphaSubCycles, U-63  $\texttt{nBufferCellsNoExtrude}, \, \underbrace{\text{U-152}}$ nCellsBetweenLevels, U-148 nFaces, U-130

nFinestSweeps, U-123nGrow, U-152 nLayerIter, U-152 nMoles, U-181 nPostSweeps, U-123 nPreSweeps, U-123 nRelaxIter, U-150, U-152 nRelaxedIter, U-152 nSmoothNormals, U-152 nSmoothPatch, U-150 nSmoothScale, U-153 nSmoothSurfaceNormals, U-152 nSmoothThickness, U-152 nSolveIter, U-150 neighbourPatch, U-142 numberOfSubdomains, U-83 n, U-83 object, U-105 order, U-83 pRefCell, U-25, U-125 pRefValue, U-25, U-125 p\_rhgRefCell, U-125 p\_rhgRefValue, U-125 patchMap, U-160 patches, U-138 preconditioner, U-121, U-122 pressure, U-51 printCeoffs, U-183 printCoeffs, U-43 processorWeights, U-82 processorWeights, U-83 purgeWrite, U-112 refGradient, U-136 refinementRegions, U-148, U-150 refinementSurfaces, U-148 refinementRegions, U-149 regions, U-60 relTol, U-54, U-121  $\texttt{relativeSizes}, \, \underline{\text{U-152}}$ relaxed, U-153resolveFeatureAngle, U-148 roots, U-83, U-84 runTimeModifiable, U-112 scotchCoeffs, U-83 setFormat, U-174 sets, U-174 simpleGrading, U-140 simulationType, U-42, U-61, U-183 smoother, U-123 snGradSchemes, U-114 snapControls, U-146 snap, U-146 solvers, U-120 solver, U-54, U-121

specie, U-181 spline, U-138 startFace, U-130 startFrom, U-24, U-111 startTime, U-24, U-111 stopAt, U-111 strategy, U-82, U-83 surfaceFormat, U-174 surfaces, U-174 thermoType, U-179 thermodynamics, U-181 timeFormat, U-112 timePrecision, U-112timeScheme, U-114 tolerance, U-54, U-121, U-150 topoSetSource, U-60 traction, U-51 transport, U-181 turbulence, U-183 type, U-132, U-133 uniform, U-175 valueFraction, U-136 value, U-23, U-136 version, U-105 vertices, U-22, U-138, U-139 writeCompression, U-112 writeControl, U-24, U-62, U-111 writeFormat, U-56, U-112 writeInterval, U-24, U-33, U-111 writePrecision, U-112 <LESModel>Coeffs, U-184 <RASModel>Coeffs, U-183 <delta>Coeffs, U-184 keyword entry CrankNicholson, U-119 CrossPowerLaw, U-61 DICGaussSeidel, U-122 DIC, U-122 DILU, U-122 Euler, U-119 FDIC, U-122 GAMG, U-54, U-121, U-122 Gamma, U-116 GaussSeidel, U-122 Gauss, U-117 LESModel, U-42, U-183 MGridGen, U-123 MUSCL, U-116 Newtonian, U-61 PBiCG, U-121 PCG, U-121 QUICK, U-119 RASModel, U-42, U-183

SFCD, U-116, U-119

**P-83** 

UMIST, U-115 adjustableRunTime, U-62, U-111 arc, U-139 ascii, U-112 backward, U-119 binary, U-112 blocking, U-80 bounded, U-117, U-118 cellLimited, U-117 cellPointFace, U-174 cellPoint, U-174 cell, U-174 clockTime, U-111 compressed, U-112 corrected, U-117, U-118 cpuTime, U-111 cubicCorrected, U-119 cubicCorrection, U-116 cyclic, U-134 diagonal, U-121, U-122 distance, U-149, U-175 dx, U-174 empty, U-134 faceAreaPair, U-122 faceLimited, U-117filteredLinear2, U-116 fixed, U-112 foamFile, U-174 fourth, U-117, U-118 general, U-112 gnuplot, U-112, U-174 hierarchical, U-82, U-83 inotifyMaster, U-80 inotify, U-80 inside, U-149 jplot, U-112, U-174 laminar, U-42, U-183 latestTime, U-111 leastSquares, U-117 limitedCubic, U-116 limitedLinear, U-116 limited, U-117, U-118 linearUpwind, U-116, U-119 linear, U-116, U-119 line, U-139 localEuler, U-119 manual, U-82, U-83 metis, U-83 midPoint, U-116 nextWrite, U-111 noWriteNow, U-111 nonBlocking, U-80 none, U-115, U-122 null, U-174

outside, U-149 patch, U-134, U-176 polyLine, U-139 polySpline, U-139 processor, U-134 raw, U-112, U-174 runTime, U-33, U-111 scheduled, U-80 scientific, U-112 scotch, U-82, U-83 simpleSpline, U-139 simple, U-82, U-83 skewLinear, U-116, U-119 smoothSolver, U-121 startTime, U-24, U-111 steadyState, U-119 stl, U-174 symmetryPlane, U-134 timeStampMaster, U-80 timeStamp, U-80 timeStep, U-24, U-33, U-111 uncompressed, U-112 uncorrected, U-117, U-118 upwind, U-116, U-119 vanLeer, U-116 vtk, U-174 wall, U-134 wedge, U-134 writeControl, U-111 writeNow, U-111 xmgr, U-112, U-174 xyz, U-175 x. U-175 y, U-175 z, U-175 kivaToFoam utility, U-91 kOmega model, U-100 kOmegaSST model, U-100, U-101 kOmegaSSTSAS model, U-101 Kronecker delta, P-20

## $\mathbf{L}$

lagrangian
 library, U-98
lagrangianIntermediate
 library, U-98
Lambda2 utility, U-93
LamBremhorstKE model, U-101
laminar model, U-100, U-101
laminar
 keyword entry, U-42, U-183
laminarFlameSpeedModels
 library, U-99
laplaceFilter model, U-101

Laplacian, P-38 laplacian, P-37 laplacian fvc member function, P-37 fvm member function, P-37 laplacianFoam solver, U-85 laplacianSchemes keyword, U-114 latestTime keyword entry, U-111 latestTime keyword, U-40 LaunderGibsonRSTM model, U-101 LaunderSharmaKE model, U-101 layers keyword, U-152 leastSquares keyword entry, U-117 leastSquares keyword, U-53 LESdeltas library, U-101 LESfilters library, U-101 LESModel keyword entry, U-42, U-183 LESModel keyword, U-184 **LESProperties** dictionary, U-183 levels keyword, U-150 libraries, U-69 library Chung, U-99 LESdeltas, U-101 LESfilters, U-101 MGridGenGAMGAgglomeration, U-98 ODE, U-97 OSspecific, U-98 OpenFOAM, U-96 P1, U-99 PV3FoamReader, U-163 PVFoamReader, U-163 SLGThermo, U-100 Wallis, U-99 autoMesh. U-97 barotropicCompressibilityModels, U-99 basicSolidThermo, U-100 basicThermophysicalModels, U-98 blockMesh, U-97 chemistryModel, U-100 coalCombustion, U-98 compressibleLESModels, U-102 compressibleRASModels, U-101 conversion, U-98 decompositionMethods, U-98 dieselSpray, U-98 distributionModels, U-98 dsmc, U-98

dynamicFvMesh, U-97 dynamicMesh, U-97edgeMesh, U-97 engine, U-98 fieldFunctionObjects, U-97 fileFormats, U-98 finiteVolume, U-97 foamCalcFunctions, U-97 forces, U-97 fvDOM, U-99 fvMotionSolvers, U-97 genericFvPatchField, U-98 incompressibleLESModels, U-101 incompressibleRASModels, U-100 incompressibleTransportModels, P-54, U-102 incompressibleTurbulenceModels, P-54 interfaceProperties, U-102 jobControl, U-97 lagrangianIntermediate, U-98 lagrangian, U-98 laminarFlameSpeedModels, U-99 linear, U-99 liquidMixtureProperties, U-100 liquidProperties, U-100 meshTools, U-97 molecularMeasurements, U-98 molecule, U-98 pairPatchAgglomeration, U-98 postCalc, U-97 potential, U-98 primitive, P-23 radiationModels, U-99randomProcesses, U-98 reactionThermophysicalModels, U-99 sampling, U-97 solidMixtureProperties, U-100 solidParticle, U-98 solidProperties, U-100 solid, U-100 specie, U-100 surfMesh. U-97 surfaceFilmModels, U-102 systemCall, U-97 thermalPorousZone, U-100 thermophysicalFunctions, U-100 thermophysical, U-179 topoChangerFvMesh, U-98 triSurface, U-97 twoPhaseInterfaceProperties, U-102 utilityFunctionObjects, U-97 viewFactor, U-99 vtkFoam, U-163 vtkPV3Foam, U-163 libs keyword, U-80, U-112

lid-driven cavity flow, U-19 LienCubicKE model, U-101 LienCubicKELowRe model, U-101 LienLeschzinerLowRe model, U-101 Lights window panel, U-167 limited keyword entry, U-117, U-118 limitedCubic keyword entry, U-116 limitedLinear keyword entry, U-116 line keyword entry, U-139 Line Style menu, U-36 linear library, U-99 linear keyword entry, U-116, U-119 linearUpwind keyword entry, U-116, U-119 liquid electrically-conducting, P-67 liquidMixtureProperties library, U-100 liquidProperties library, U-100 lists, P-29 List<Type> template class, P-29 localEuler keyword entry, U-119 location keyword, U-105 locationInMesh keyword, U-148, U-149 locDynOneEqEddy model, U-101 lowCpCoeffs keyword, U-182 lowReOneEqEddy model, U-102 LRDDiffStress model, U-102 LRR model, U-101 lsGrad fvc member function, P-37 LTSInterFoam solver, U-87 LTSReactingParcelFoam solver, U-88

#### $\mathbf{M}$

Mach utility, U-93 mag tensor member function, P-25 magneticFoam solver, U-89 magnetohydrodynamics, P-67 magSqr tensor member function, P-25 *Make* directory, U-73 make script/alias, U-71 *Make/files* file, U-74 manual

keyword entry, U-82, U-83 manualCoeffs keyword, U-83 mapFields utility, U-32, U-39, U-43, U-56, U-90, U-160 mapping fields, U-160 Marker Style menu, U-36 matrices tools, U-97 max tensor member function, P-25 maxAlphaCo keyword, U-62 maxBoundarySkewness keyword, U-153 maxCo keyword, U-62 maxConcave keyword, U-153 maxDeltaT keyword, U-62 maxDeltaxyz model, U-101 maxFaceThicknessRatio keyword, U-152 maxGlobalCells keyword, U-148 maximum iterations, U-121 maxInternalSkewness keyword, U-153 maxIter keyword, U-121 maxLocalCells keyword, U-148 maxNonOrtho keyword, U-153 maxThicknessToMedialRatio keyword, U-152 mdEquilibrationFoam solver, U-89 mdFoam solver, U-89 mdInitialise utility, U-90 mechanicalProperties dictionary, U-51 memory tools, U-97 menu Color By, U-166 Current Time Controls, U-27, U-165 Edit, U-167, U-168 Help, U-167 Line Style, U-36 Marker Style, U-36 VCR Controls, U-27, U-165 View, U-167 menu entry Plot Over Line, U-35 Save Animation, U-169 Save Screenshot, U-169 Settings, U-168 Show Color Legend, U-27 Solid Color, U-166 Toolbars, U-167 View Settings..., U-26 View Settings, U-26, U-167 Wireframe, U-166 fvSchemes, U-53 mergeLevels keyword, U-123 mergeMeshes utility, U-91 mergeOrSplitBaffles utility, U-91

mergePatchPairs keyword, U-138 mergeTolerance keyword, U-146 mesh 1-dimensional, U-130 1D, U-130 2-dimensional, U-130 2D, U-130 axi-symmetric, U-130 basic, P-31 block structured, U-136 decomposition, U-81 description, U-127 finite volume, P-31 generation, U-136, U-145 grading, U-136, U-140 grading, example of, P-53 non-orthogonal, P-45 refinement, P-62 resolution, U-32 specification, U-127 split-hex, U-145 Stereolithography (STL), U-145 surface, U-145 validity constraints, U-127 Mesh Parts window panel, U-25 meshes tools, U-97 meshQualityControls keyword, U-146 meshTools library, U-97 message passing interface openMPI, U-82 method keyword, U-83 metis keyword entry, U-83 MGridGenGAMGAgglomeration library, U-98 MGridGen keyword entry, U-123 mhdFoam solver, P-69, U-89 midPoint keyword entry, U-116 midPoint keyword, U-175 midPointAndFace keyword, U-175 min tensor member function, P-25 minArea keyword, U-153 minDeterminant keyword, U-153 minFaceWeight keyword, U-153 minFlatness keyword, U-153 minMedianAxisAngle keyword, U-152 MINMOD differencing, P-38 minRefinementCells keyword, U-148 minThickness keyword, U-152 minTriangleTwist keyword, U-153

minTwist keyword, U-153 minVol keyword, U-153 minVolRatio keyword, U-153 mirrorMesh utility, U-91 mixed boundary condition, U-136 mixedSmagorinsky model, U-101 mixtureAdiabaticFlameT utility, U-96 mode keyword, U-149 model APIfunctions, U-100 BirdCarreau, U-102 CrossPowerLaw, U-102 DeardorffDiffStress, U-102 GuldersEGRLaminarFlameSpeed, U-99 GuldersLaminarFlameSpeed, U-99 HerschelBulkley, U-102 LRDDiffStress, U-102 LRR, U-101 LamBremhorstKE, U-101 LaunderGibsonRSTM, U-101 LaunderSharmaKE, U-101 LienCubicKELowRe, U-101 LienCubicKE, U-101 LienLeschzinerLowRe, U-101 NSRDSfunctions, U-100 Newtonian, U-102 NonlinearKEShih, U-101 PrandtlDelta, U-101 RNGkEpsilon, U-101 Smagorinsky2, U-101 Smagorinsky, U-101, U-102 SpalartAllmarasDDES, U-102 SpalartAllmarasIDDES, U-102 SpalartAllmaras, U-101, U-102 anisotropicFilter, U-101 basicMultiComponentMixture, U-99, U-180 chemistryModel, U-100 chemistrySolver, U-100 constLaminarFlameSpeed, U-99 constTransport, U-100, U-180 cubeRootVolDelta, U-101 dieselMixture, U-99, U-180 distributed, U-98 dynLagrangian, U-101 dynMixedSmagorinsky, U-101 dynOneEqEddy, U-101, U-102 dynSmagorinsky, U-101 eConstThermo, U-100, U-179 ePsiThermo, U-98, U-180 egrMixture, U-99, U-180 hConstThermo, U-100, U-179 hPolynomialThermo, U-100, U-179 hPsiMixtureThermo, U-99, U-180

hPsiThermo, U-98, U-180 hRhoMixtureThermo, U-99, U-180 hRhoThermo, U-98, U-180 hhuMixtureThermo, U-99, U-180 homogenousDynSmagorinsky, U-101 homogeneousMixture, U-99, U-180 hsPsiMixtureThermo, U-99, U-180 hsPsiThermo, U-98, U-180 hsRhoMixtureThermo, U-99, U-180 icoPolynomial, U-100, U-179 inhomogeneousMixture, U-99, U-180 interfaceProperties, U-102 janafThermo, U-100, U-179 kEpsilon, U-100, U-101 kOmegaSSTSAS, U-101 kOmegaSST, U-100, U-101 kOmega, U-100 laminar, U-100, U-101 laplaceFilter, U-101 locDynOneEqEddy, U-101 lowReOneEqEddy, U-102 maxDeltaxyz, U-101 mixedSmagorinsky, U-101 multiComponentMixture, U-99, U-180 oneEqEddy, U-101, U-102 perfectGas, U-100, U-179 polynomialTransport, U-100, U-180 powerLaw, U-102 ptsotchDecomp, U-98 pureMixture, U-99, U-180 qZeta, U-101 reactingMixture, U-99, U-180 realizableKE, U-101 reconstruct, U-98 scaleSimilarity, U-101 scotchDecomp, U-98 simpleFilter, U-101 smoothDelta, U-101 specieThermo, U-100, U-180 spectEddyVisc, U-102sutherlandTransport, U-100, U-180 veryInhomogeneousMixture, U-99, U-180 modifyMesh utility, U-92 molecularMeasurements library, U-98 molecule library, U-98 molWeight keyword, U-181 moveDynamicMesh utility, U-91 moveEngineMesh utility, U-91 moveMesh utility, U-91 movingWallVelocity boundary condition, U-137 MPI

openMPI, U-82 MRFInterFoam solver, U-87 MRFMultiphaseInterFoam solver, U-87 MRFSimpleFoam solver, U-85 mshToFoam utility, U-91 mu keyword, U-182 multiComponentMixture model, U-99, U-180 multigrid geometric-algebraic, U-122 multiphaseInterFoam solver, U-87 MUSCL keyword entry, U-116

## Ν

n keyword, U-83 nabla operator, P-27 nAlphaSubCycles keyword, U-63 nBufferCellsNoExtrude keyword, U-152 nCellsBetweenLevels keyword, U-148 neighbour dictionary, U-129 neighbourPatch keyword, U-142 netgenNeutralToFoam utility, U-91 Newtonian keyword entry, U-61 Newtonian model, U-102 nextWrite keyword entry, U-111 nFaces keyword, U-130 nFinestSweeps keyword, U-123 nGrow keyword, U-152 nLayerIter keyword, U-152 nMoles keyword, U-181 non-orthogonal mesh, P-45 nonBlocking keyword entry, U-80 none keyword entry, U-115, U-122 NonlinearKEShih model, U-101 nonNewtonianIcoFoam solver, U-86 noWriteNow keyword entry, U-111 nPostSweeps keyword, U-123 nPreSweeps keyword, U-123 nRelaxedIter keyword, U-152 nRelaxIter keyword, U-150, U-152 nSmoothNormals keyword, U-152 nSmoothPatch keyword, U-150 nSmoothScale keyword, U-153 nSmoothSurfaceNormals keyword, U-152 nSmoothThickness keyword, U-152 nSolveIter keyword, U-150 NSRDSfunctions model, U-100

null keyword entry, U-174 numberOfSubdomains keyword, U-83

#### 0

object keyword, U-105 objToVTK utility, U-91 ODE library, U-97 oneEqEddy model, U-101, U-102 Opacity text box, U-167 **OpenFOAM** applications, U-69 file format, U-104libraries, U-69 **OpenFOAM** library, U-96 OpenFOAM file syntax //, U-104 openMPI message passing interface, U-82 MPI, U-82 operator scalar, P-28 vector, P-27 Options window, U-168 options file, U-73 order keyword, U-83 Orientation Axes button, U-26, U-167 OSspecific library, U-98 outer product, see tensor, outer product outlet boundary condition, P-68 outletInlet boundary condition, U-137 outside keyword entry, U-149 owner dictionary, U-129

#### Ρ

p field, U-24
P1
 library, U-99
p\_rhgRefCell keyword, U-125
p\_rhgRefValue keyword, U-125
pairPatchAgglomeration
 library, U-98
paraFoam, U-25, U-163
parallel
 running, U-81
partialSlip
 boundary condition, U-137

particleTracks utility, U-94 patch boundary condition, U-135 patch keyword entry, U-134, U-176 patchAverage utility, U-93 patches keyword, U-138 patchIntegrate utility, U-94 patchMap keyword, U-160 patchSummary utility, U-96 PBiCG keyword entry, U-121 PCG keyword entry, U-121 pdfPlot utility, U-94 PDRFoam solver, U-88 PDRMesh utility, U-92 Pe utility, U-93 perfectGas model, U-100, U-179 permutation symbol, P-19 pimpleDyMFoam solver, U-86 pimpleFoam solver, U-86 Pipeline Browser window, U-25, U-164 PISO dictionary, U-25 pisoFoam solver, U-19, U-86 Plot Over Line menu entry, U-35 plot3dToFoam utility, U-91 pointField class, P-31 pointField<Type> template class, P-33 points dictionary, U-129, U-136 polyBoundaryMesh class, P-31 polyDualMesh utility, U-91 polyLine keyword entry, U-139 polyMesh directory, U-104, U-129 polyMesh class, P-31, U-127, U-129 polynomialTransport model, U-100, U-180 polyPatch class, P-31 polyPatchList class, P-31 polySpline keyword entry, U-139 porousExplicitSourceReactingParcelFoam solver, U-89 porousInterFoam solver, U-87 porousSimpleFoam solver, U-86 post-processing, U-163 post-processing paraFoam, U-163 postCalc library, U-97 postChannel utility, U-94

potential library, U-98 potentialFoam solver, P-46, U-85 pow tensor member function, P-25 powerLaw model, U-102 pPrime2 utility, U-93 Pr keyword, U-182 PrandtlDelta model, U-101 preconditioner keyword, U-121, U-122 pRefCell keyword, U-25, U-125 pRefValue keyword, U-25, U-125 pressure keyword, U-51 pressure waves in liquids, P-62 pressureDirectedInletVelocity boundary condition, U-137 pressureInletVelocity boundary condition, U-137 pressureOutlet boundary condition, P-63 pressureTransmissive boundary condition, U-137 primitive library, P-23 primitives tools, U-97 printCeoffs keyword, U-183 printCoeffs keyword, U-43 processorWeights keyword, U-82 probeLocations utility, U-94 process background, U-26, U-81 foreground, U-26 processor boundary condition, U-135 processor keyword entry, U-134 processorN directory, U-82 processorWeights keyword, U-83 Properties window panel, U-27, U-164 ptot utility, U-94 ptsotchDecomp model, U-98 pureMixture model, U-99, U-180 purgeWrite keyword, U-112 PV3FoamReader library, U-163 **PVFoamReader** library, U-163

## $\mathbf{Q}$

Q utility, U-93 QUICK keyword entry, U-119 qZeta model, U-101

## R

R utility, U-93 radiationModels library, U-99 randomProcesses library, U-98 RASModel keyword entry, U-42, U-183 RASModel keyword, U-183 raw keyword entry, U-112, U-174 reactingFoam solver, U-88 reactingMixture model, U-99, U-180 reactingParcelFilmFoam solver, U-89 reactingParcelFoam solver, U-89 reactionThermophysicalModels library, U-99 realizableKE model, U-101 reconstruct model, U-98 reconstructPar utility, U-85, U-95 reconstructParMesh utility, U-95 redistributeMeshPar utility, U-96 refGradient keyword, U-136 refineHexMesh utility, U-92 refinementRegions keyword, U-149 refinementLevel utility, U-92 refinementRegions keyword, U-148, U-150 refinementSurfaces keyword, U-148 refineMesh utility, U-91 refineWallLayer utility, U-92 Refresh Times button, U-27 regions keyword, U-60 relative tolerance, U-121 relativeSizes keyword, U-152 relaxed keyword, U-153 relTol keyword, U-54, U-121 removeFaces utility, U-92 Render View window, U-168 Render View window panel, U-168 renumberMesh utility, U-91 Rescale to Data Range button, U-27 Reset button, U-164 resolveFeatureAngle keyword, U-148 restart, U-40Reynolds number, U-19, U-23 rhoPorousMRFLTSPimpleFoam solver, U-86 rhoPorousMRFPimpleFoam solver, U-86 rhoPorousMRFSimpleFoam solver, U-86 rhoCentralDyMFoam solver, U-86 rhoCentralFoam solver, U-86 rhoPimpleFoam solver, U-86 rhoReactingFoam solver, U-88 rhoSimpleFoam solver, U-86 rhoSimplecFoam solver, U-86

rmdepall script/alias, U-76 RNGkEpsilon model, U-101 roots keyword, U-83, U-84 rotateMesh utility, U-91 run parallel, U-81 run directory, U-103 runTime keyword entry, U-33, U-111 runTimeModifiable keyword, U-112

#### $\mathbf{S}$

sammToFoam utility, U-91 sample utility, U-94, U-173 sampling library, U-97 Save Animation menu entry, U-169 Save Screenshot menu entry, U-169 scalar, P-16 operator, P-28 scalar class, P-23 scalarField class, P-29 scalarTransportFoam solver, U-85 scale tensor member function, P-25 scalePoints utility, U-157 scaleSimilarity model, U-101 scheduled keyword entry, U-80 scientific keyword entry, U-112 scotch keyword entry, U-82, U-83 scotchCoeffs keyword, U-83 scotchDecomp model, U-98 script/alias foamCorrectVrt, U-158 foamJob, U-177 foamLog, U-177 make, U-71 rmdepall, U-76 wclean, U-75 wmake, U-71 second time derivative, P-37 Seed window, U-169 selectCells utility, U-92 Set Ambient Color button, U-166 setFields utility, U-60, U-90 setFormat keyword, U-174 sets keyword, U-174 setSet utility, U-91 setsToZones utility, U-92

Settings menu entry, U-168 settlingFoam solver, U-87 SFCD keyword entry, U-116, U-119 shallowWaterFoam solver, U-86 shape, U-140Show Color Legend menu entry, U-27 SI units, U-107 simple keyword entry, U-82, U-83 simpleFilter model, U-101 simpleFoam solver, P-54, U-86 simpleGrading keyword, U-140 simpleSpline keyword entry, U-139 simulationType keyword, U-42, U-61, U-183 singleCellMesh utility, U-92 skew tensor member function, P-25 skewLinear keyword entry, U-116, U-119 SLGThermo library, U-100 slice class, P-31 slip boundary condition, U-137 Smagorinsky model, U-101, U-102 Smagorinsky2 model, U-101 smapToFoam utility, U-93 smoothDelta model, U-101 smoother keyword, U-123 smoothSolver keyword entry, U-121 snap keyword, U-146 snapControls keyword, U-146 snappyHexMesh utility background mesh, U-146 cell removal, U-149 cell splitting, U-147 mesh layers, U-150 meshing process, U-145 snapping to surfaces, U-150 snappyHexMesh utility, U-90, U-145 snappyHexMeshDict file, U-145 snGrad fvc member function, P-37 snGradCorrection fvc member function, P-37 snGradSchemes keyword, U-114 solid library, U-100 Solid Color

menu entry, U-166 solidDisplacementFoam solver, U-89 solidDisplacementFoam solver, U-52 solidEquilibriumDisplacementFoam solver, U-89 solidMixtureProperties library, U-100 solidParticle library, U-98 solidProperties library, U-100 solver LTSInterFoam, U-87 LTSReactingParcelFoam, U-88 MRFInterFoam, U-87 MRFMultiphaseInterFoam, U-87 MRFSimpleFoam, U-85 PDRFoam, U-88 SRFSimpleFoam, U-86 XiFoam, U-88 adjointShapeOptimizationFoam, U-85 blockMesh, P-47 boundaryFoam, U-85 bubbleFoam, U-87 buoyantBaffleSimpleFoam, U-88 buoyantBoussinesgPimpleFoam, U-88 buoyantBoussinesqSimpleFoam, U-88 buoyantPimpleFoam, U-88 buoyantSimpleFoam, U-88 buovantSimpleRadiationFoam, U-88 cavitatingFoam, U-87 channelFoam, U-85 chemFoam, U-88 chtMultiRegionFoam, U-88 coalChemistryFoam, U-88 coldEngineFoam, U-88 compressibleInterFoam, U-87 dieselEngineFoam, U-88 dieselFoam, U-88 dnsFoam, U-87 dsmcFoam, U-89 electrostaticFoam, U-89 engineFoam, U-88 financialFoam, U-89 fireFoam, U-88 icoFoam, U-19, U-23, U-24, U-26, U-85 icoUncoupledKinematicParcelDyMFoam, U-88 icoUncoupledKinematicParcelFoam, U-88 interDyMFoam, U-87 interFoam, U-87 interMixingFoam, U-87 interPhaseChangeFoam, U-87 laplacianFoam, U-85 magneticFoam, U-89

mdEquilibrationFoam, U-89 mdFoam, U-89 mhdFoam, P-69, U-89 multiphaseInterFoam, U-87 nonNewtonianIcoFoam, U-86 pimpleDyMFoam, U-86 pimpleFoam, U-86 pisoFoam, U-19, U-86 porousExplicitSourceReactingParcelFoam, U-89 porousInterFoam, U-87 porousSimpleFoam, U-86 potentialFoam, P-46, U-85 reactingFoam, U-88 reactingParcelFilmFoam, U-89 reactingParcelFoam, U-89 rhoCentralDyMFoam, U-86 rhoCentralFoam, U-86 rhoPimpleFoam, U-86 rhoReactingFoam, U-88 rhoSimpleFoam, U-86 rhoSimplecFoam, U-86 rhoPorousMRFLTSPimpleFoam, U-86 rhoPorousMRFPimpleFoam, U-86 rhoPorousMRFSimpleFoam, U-86 scalarTransportFoam, U-85 settlingFoam, U-87 shallowWaterFoam, U-86 simpleFoam, P-54, U-86 solidDisplacementFoam, U-89 solidDisplacementFoam, U-52 solidEquilibriumDisplacementFoam, U-89 sonicDyMFoam, U-86 sonicFoam, P-59, U-86 sonicLiquidFoam, P-63, U-86 twoLiquidMixingFoam, U-87 twoPhaseEulerFoam, U-87 uncoupledKinematicParcelFoam, U-89 windSimpleFoam, U-86 solver keyword, U-54, U-121 solver relative tolerance, U-121 solver tolerance, U-121 solvers keyword, U-120 sonicDyMFoam solver, U-86 sonicFoam solver, P-59, U-86 sonicLiquidFoam solver, P-63, U-86 source, P-37 SpalartAllmaras model, U-101, U-102 SpalartAllmarasDDES model, U-102 SpalartAllmarasIDDES model, U-102 specie library, U-100 specie keyword, U-181 specieThermo model, U-100, U-180

spectEddyVisc model, U-102 spline keyword, U-138 splitCells utility, U-92 splitMesh utility, U-92 splitMeshRegions utility, U-92 sqr tensor member function, P-25 sqrGradGrad fvc member function, P-37 SRFSimpleFoam solver, U-86 star3ToFoam utility, U-91 star4ToFoam utility, U-91 startFace keyword, U-130 startFrom keyword, U-24, U-111 starToFoam utility, U-154 startTime keyword entry, U-24, U-111 startTime keyword, U-24, U-111 steady flow turbulent, P-53 steadyParticleTracks utility, U-94 steadyState keyword entry, U-119 Stereolithography (STL), U-145 stitchMesh utility, U-92 stl keyword entry, U-174 stopAt keyword, U-111 strategy keyword, U-82, U-83 streamFunction utility, U-93 stress analysis of plate with hole, U-47stressComponents utility, U-93 Style window panel, U-26, U-166 Su fvm member function, P-37 subsetMesh utility, U-92 summation convention, P-17 SUPERBEE differencing, P-38 supersonic flow, P-58 supersonic flow over forward step, P-58 supersonicFreeStream boundary condition, U-137 surface mesh, U-145 surfaceAdd utility, U-94 surfaceAutoPatch utility, U-94 surfaceCheck utility, U-94 surfaceClean utility, U-94 surfaceCoarsen utility, U-94 surfaceConvert utility, U-94 surfaceFeatureConvert utility, U-94 surfaceFeatureExtract utility, U-94, U-148 surfaceField<Type> template class, P-33 surfaceFilmModels library, U-102

surfaceFind utility, U-94 surfaceFormat keyword, U-174 surfaceInertia utility, U-95 surfaceMesh tools, U-97 surfaceMeshConvert utility, U-95 surfaceMeshConvertTesting utility, U-95 surfaceMeshExport utility, U-95 surfaceMeshImport utility, U-95 surfaceMeshInfo utility, U-95 surfaceMeshTriangulate utility, U-95 surfaceNormalFixedValue boundary condition, U-137 surfaceOrient utility, U-95 surfacePointMerge utility, U-95 surfaceRedistributePar utility, U-95 surfaceRefineRedGreen utility, U-95 surfaces keyword, U-174 surfaceSmooth utility, U-95 surfaceSplitByPatch utility, U-95 surfaceSplitNonManifolds utility, U-95 surfaceSubset utility, U-95 surfaceToPatch utility, U-95 surfaceTransformPoints utility, U-95 surfMesh library, U-97 SuSp fvm member function, P-37 sutherlandTransport model, U-100, U-180 symm tensor member function, P-25 symmetryPlane boundary condition, P-63, U-135 symmetryPlane keyword entry, U-134 symmTensorField class, P-29 symmTensorThirdField class, P-29 system directory, P-50, U-104 systemCall library, U-97

## $\mathbf{T}$

T() tensor member function, P-25 Tcommon keyword, U-182 template class GeometricBoundaryField, P-32 fvMatrix, P-33 dimensioned<Type>, P-25 FieldField<Type>, P-32 Field<Type>, P-29 geometricField<Type>, P-32 List<Type>, P-29 pointField<Type>, P-33 surfaceField<Type>, P-33

volField<Type>, P-33 temporal discretisation, P-42 Crank Nicholson, P-42 Euler implicit, P-42 explicit, P-42 in OpenFOAM, P-43 tensor, P-15 addition, P-17 algebraic operations, P-17 algebraic operations in OpenFOAM, P-23 antisymmetric, see tensor, skew calculus, P-27 classes in OpenFOAM, P-23 cofactors, P-22 component average, P-20 component maximum, P-20 component minimum, P-20 determinant, P-22 deviatoric, P-21 diagonal, P-21 dimension, P-16 double inner product, P-19 geometric transformation, P-20 Hodge dual, P-22 hydrostatic, P-21 identities, P-21 identity, P-20inner product, P-18 inverse, P-22 magnitude, P-20 magnitude squared, P-20 mathematics, P-15 notation. P-17 nth power, P-20 outer product, P-19 rank, P-16 rank 3, P-16 scalar division, P-18 scalar multiplication, P-17 scale function, P-20 second rank. P-16 skew, P-21 square of, P-20 subtraction, P-17 symmetric, P-21 symmetric rank 2, P-16 symmetric rank 3, P-16 trace, P-21 transformation, P-20 transpose, P-16, P-21 triple inner product, P-19 vector cross product, P-19 tensor class, P-23 tensor member function

\*. P-25 +, P-25 -, P-25 /. P-25 &, P-25 &&, P-25 ^, P-25 cmptAv, P-25 cofactors, P-25 det, P-25 dev, P-25 diag, P-25 I, P-25 inv, P-25 mag, P-25 magSqr, P-25 max. P-25min, P-25 pow, P-25 scale, P-25 skew, P-25 sqr, P-25 symm, P-25 T(), P-25 tr, P-25 transform, P-25 tensorField class, P-29 tensorThirdField class, P-29 tetgenToFoam utility, U-91 text box Opacity, U-167 thermalPorousZone library, U-100 thermalProperties dictionary, U-52 thermodynamics keyword, U-181 thermophysical library, U-179 thermophysicalFunctions library, U-100 thermophysicalProperties dictionary, U-179 thermoType keyword, U-179 Thigh keyword, U-182 time control, U-111 time derivative, P-37 first, P-39 second, P-37, P-39 time step, U-24 timeFormat keyword, U-112 timePrecision keyword, U-112 timeScheme keyword, U-114 timeStamp

#### P-94

keyword entry, U-80 timeStampMaster keyword entry, U-80 timeStep keyword entry, U-24, U-33, U-111 Tlow keyword, U-182 tolerance solver, U-121 solver relative, U-121 tolerance keyword, U-54, U-121, U-150 Toolbars menu entry, U-167 tools algorithms, U-96 cfdTools, U-97 containers, U-96 db, U-96 dimensionSet, U-97 dimensionedTypes, U-97 fields, U-97 finiteVolume, U-97 fvMatrices, U-97 fvMesh, U-97 global, U-97graph, U-97 interpolations, U-97 interpolation, U-97 matrices, U-97 memory, U-97 meshes, U-97 primitives, U-97 surfaceMesh, U-97 volMesh, U-97 topoChangerFvMesh library, U-98 topoSet utility, U-92 topoSetSource keyword, U-60 totalPressure boundary condition, U-137 tr tensor member function. P-25 trace, see tensor, trace traction keyword, U-51 transform tensor member function, P-25transformPoints utility, U-92 transport keyword, U-181 transportProperties dictionary, U-23, U-40, U-43 transportProperties file, U-61 triple inner product, P-19 triSurface library, U-97 Ts keyword, U-182

turbulence dissipation, U-41 kinetic energy, U-41 length scale, U-42 turbulence keyword, U-183 turbulence model RAS, U-41 turbulenceProperties dictionary, U-42, U-61, U-183 turbulent flow steady, P-53 turbulentInlet boundary condition, U-137 tutorials breaking of a dam, U-57 lid-driven cavity flow, U-19 stress analysis of plate with hole, U-47 tutorials directory, P-45, U-19 twoLiquidMixingFoam solver, U-87 twoPhaseEulerFoam solver, U-87 twoPhaseInterfaceProperties library, U-102 type keyword, U-132, U-133

## U

U field, U-24 Ucomponents utility, P-70 UMIST keyword entry, U-115 uncompressed keyword entry, U-112 uncorrected keyword entry, U-117, U-118 uncoupledKinematicParcelFoam solver, U-89 uniform keyword, U-175 units base, U-107 of measurement, P-25, U-107 S.I. base, P-25 SI, U-107 Système International, U-107 United States Customary System, U-107 USCS, U-107 Update GUI button, U-165 uprime utility, U-93 upwind keyword entry, U-116, U-119 upwind differencing, P-38, U-62 USCS units, U-107 Use Parallel Projection button, U-26 Use parallel projection button, U-167 utility Co. U-93 Lambda2, U-93

Mach, U-93 PDRMesh, U-92 Pe, U-93 Q, U-93 R, U-93 Ucomponents, P-70 adiabaticFlameT, U-96 ansysToFoam, U-90 applyBoundaryLayer, U-90 applyWallFunctionBoundaryConditions, **U-90** attachMesh, U-91 autoPatch, U-91 autoRefineMesh, U-92 blockMesh, U-39, U-90, U-136 boxTurb, U-90 cfx4ToFoam, U-90, U-154 changeDictionary, U-90 checkMesh, U-91, U-155 chemkinToFoam, U-96 collapseEdges, U-92 combinePatchFaces, U-92 createBaffles, U-91 createPatch, U-91 createTurbulenceFields, U-93 datToFoam, U-90 decomposePar, U-81, U-82, U-95 deformedGeom, U-91 dsmcFieldsCalc, U-94 dsmcInitialise, U-90 engineCompRatio, U-94 engineSwirl, U-90 ensight74FoamExec, U-172 ensightFoamReader, U-92 enstrophy, U-93 equilibriumCO, U-96 equilibriumFlameT, U-96 execFlowFunctionObjects, U-94 expandDictionary, U-96 extrude2DMesh, U-90 extrudeMesh. U-90 extrudeToRegionMesh, U-90 faceAgglomerate, U-90 fieldview9Reader, U-92 flattenMesh, U-91 flowType, U-93 fluent3DMeshToFoam, U-90 fluentMeshToFoam, U-90, U-154 foamCalc, U-35, U-94 foamDataToFluent, U-92, U-170 foamDebugSwitches, U-96 foamFormatConvert, U-96 foamInfoExec, U-96 foamListTimes, U-94

foamMeshToFluent, U-90, U-170 foamToEnsightParts, U-92 foamToEnsight, U-92 foamToFieldview9, U-92 foamToGMV, U-92 foamToStarMesh, U-90 foamToSurface, U-91 foamToTecplot360, U-92 foamToVTK, U-92 foamUpgradeCyclics, U-90 foamUpgradeFvSolution, U-90gambitToFoam, U-91, U-154 gmshToFoam, U-91 ideasToFoam, U-154 ideasUnvToFoam, U-91 insideCells, U-91 kivaToFoam, U-91 mapFields, U-32, U-39, U-43, U-56, U-90, U-160 mdInitialise, U-90 mergeMeshes, U-91 mergeOrSplitBaffles, U-91 mirrorMesh, U-91 mixtureAdiabaticFlameT, U-96 modifyMesh, U-92 moveDynamicMesh, U-91 moveEngineMesh, U-91 moveMesh, U-91 mshToFoam, U-91 netgenNeutralToFoam, U-91 objToVTK, U-91 pPrime2, U-93 particleTracks, U-94 patchAverage, U-93 patchIntegrate, U-94 patchSummary, U-96 pdfPlot, U-94 plot3dToFoam, U-91 polyDualMesh, U-91 postChannel, U-94 probeLocations, U-94 ptot, U-94 reconstructParMesh, U-95 reconstructPar, U-85, U-95 redistributeMeshPar, U-96 refineHexMesh, U-92 refineMesh, U-91 refineWallLayer, U-92 refinementLevel, U-92 removeFaces, U-92 renumberMesh, U-91 rotateMesh, U-91 sammToFoam, U-91 sample, U-94, U-173

scalePoints, U-157 selectCells. U-92 setFields, U-60, U-90 setSet, U-91 setsToZones, U-92 singleCellMesh, U-92 smapToFoam, U-93 snappyHexMesh, U-90, U-145 splitCells, U-92 splitMeshRegions, U-92splitMesh, U-92 star3ToFoam, U-91 star4ToFoam, U-91 starToFoam, U-154 steadyParticleTracks, U-94 stitchMesh, U-92 streamFunction, U-93 stressComponents, U-93 subsetMesh, U-92 surfaceAdd, U-94 surfaceAutoPatch, U-94 surfaceCheck, U-94 surfaceClean, U-94 surfaceCoarsen, U-94 surfaceConvert, U-94 surfaceFeatureConvert, U-94 surfaceFeatureExtract, U-94, U-148 surfaceFind, U-94 surfaceInertia, U-95 surfaceMeshConvertTesting, U-95 surfaceMeshConvert, U-95 surfaceMeshExport, U-95 surfaceMeshImport, U-95 surfaceMeshInfo, U-95 surfaceMeshTriangulate, U-95 surfaceOrient, U-95 surfacePointMerge, U-95 surfaceRedistributePar, U-95 surfaceRefineRedGreen, U-95 surfaceSmooth, U-95 surfaceSplitByPatch, U-95 surfaceSplitNonManifolds, U-95 surfaceSubset, U-95 surfaceToPatch, U-95 surfaceTransformPoints, U-95 tetgenToFoam, U-91 topoSet, U-92 transformPoints, U-92 uprime, U-93 viewFactorGen, U-90 vorticity, U-93 wallFunctionTable, U-90 wallGradU, U-93 wallHeatFlux, U-93

wallShearStress, U-93 wdot, U-94 writeCellCentres, U-94 writeMeshObj, U-91 yPlusLES, U-93 yPlusRAS, U-93 zipUpMesh, U-92 utilityFunctionObjects library, U-97

#### $\mathbf{V}$

value keyword, U-23, U-136 valueFraction keyword, U-136 van Leer differencing, P-38 vanLeer keyword entry, U-116 VCR Controls menu, U-27, U-165 vector, P-16 operator, P-27 unit, P-20 vector class, P-23, U-107 vector product, see tensor, vector cross product vectorField class, P-29 version keyword, U-105 vertices keyword, U-22, U-138, U-139 veryInhomogeneousMixture model, U-99, U-180 View menu, U-167 View Settings menu entry, U-26, U-167 View Settings (Render View) window, U-167 View Settings... menu entry, U-26viewFactor library, U-99 viewFactorGen utility, U-90 viscosity kinematic, U-23, U-43 volField<Type> template class, P-33 volMesh tools, U-97 vorticity utility, U-93 vtk keyword entry, U-174 vtkFoam library, U-163 vtkPV3Foam library, U-163

#### $\mathbf{W}$

wall
 boundary condition, P-63, P-68, U-59,
 U-135
wall
 keyword entry, U-134
wallBuoyantPressure

boundary condition, U-137 wallFunctionTable utility, U-90 wallGradU utility, U-93 wallHeatFlux utility, U-93 Wallis library, U-99 wallShearStress utility, U-93 wclean script/alias, U-75 wdot utility, U-94 wedge boundary condition, U-130, U-135, U-144 wedge keyword entry, U-134 window Color Legend, U-29 Options, U-168 Pipeline Browser, U-25, U-164 Render View, U-168 Seed, U-169 View Settings (Render View), U-167 window panel Animations, U-168 Annotation, U-26, U-167 Charts, U-168 Color Legend, U-166 Color Scale, U-166 Colors, U-168 Display, U-25, U-27, U-164, U-165 General, U-167, U-168 Information, U-164 *Lights*, **U-167** Mesh Parts, U-25 Properties, U-27, U-164 Render View, U-168 Style, U-26, U-166 windSimpleFoam solver, U-86 Wireframe menu entry, U-166 WM\_ARCH environment variable, U-76 WM\_ARCH\_OPTION environment variable, U-76 WM\_COMPILE\_OPTION environment variable, U-76 WM\_COMPILER environment variable, U-76 WM\_COMPILER\_BIN environment variable, U-76 WM\_COMPILER\_DIR environment variable, U-76 WM\_COMPILER\_LIB environment variable, U-76 WM\_DIR

environment variable, U-76 WM\_MPLIB environment variable, U-76 WM\_OPTIONS environment variable, U-76 WM\_PRECISION\_OPTION environment variable, U-76 WM\_PROJECT environment variable, U-76 WM\_PROJECT\_DIR environment variable, U-76 WM\_PROJECT\_INST\_DIR environment variable, U-76 WM\_PROJECT\_USER\_DIR environment variable, U-76 WM\_PROJECT\_VERSION environment variable, U-76 wmake platforms, U-72 wmake script/alias, U-71 word class, P-25, P-31 writeCellCentres utility, U-94 writeCompression keyword, U-112 writeControl keyword entry, U-111 writeControl keyword, U-24, U-62, U-111 writeFormat keyword, U-56, U-112 writeInterval keyword, U-24, U-33, U-111 writeMeshObj utility, U-91 writeNow keyword entry, U-111 writePrecision keyword, U-112

## Χ

x

```
keyword entry, U-175
XiFoam solver, U-88
xmgr
keyword entry, U-112, U-174
xyz
keyword entry, U-175
```

## Υ

y keyword entry, U-175 yPlusLES utility, U-93 yPlusRAS utility, U-93

## $\mathbf{Z}$

z keyword entry, U-175 zeroGradient boundary condition, U-136 zipUpMesh utility, U-92