VFB

100) Apliquem la regla de Cràmer per resoldre els sistemes següents:

a) $\begin{cases} x + y - z = 1 \\ x - y + z = 1 \\ -x + y + z = 1 \end{cases}$ $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

· Tenim 3 incògnites i 3 equacions.

· $|A| = -1 - 1 - 1 + 1 - 1 - 1 = -4 \neq 0$

Per tant, podem aplicar la regla de Cràmer:

$$x = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{-4} = \frac{\cancel{-1} + \cancel{1} - 1 - 1 - 1 - 1}{-4} = \frac{-4}{-4} = 1$$

$$y = \frac{\begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{-4} = \frac{\cancel{1} - \cancel{1} - 1 - 1 - 1 - 1}{-4} = \frac{-4}{-4} = 1$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}}{-4} = \frac{-1 - 1 + \cancel{1} - \cancel{1} - 1 - 1}{-4} = \frac{-4}{-4} = 1.$$

Per tant, les solucions són $(1, 1, 1)$

b) $\begin{cases} 3x - y = 2 \\ 2x + y + z = 0 \\ 3y + 2z = -1 \end{cases}$ $A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 3 & 2 \end{pmatrix}$

$|A| = 6 + 4 - 9 = 1 \neq 0$

• 3 eq. i 3 incògnites ⇒ Podem aplicar Cràmer

$$x = \frac{\begin{vmatrix} 2 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 3 & 2 \end{vmatrix}}{1} = \frac{4+1-6}{1} = \frac{-1}{1} = -1$$

$$y = \frac{\begin{vmatrix} 3 & 2 & 0 \\ 2 & 0 & 1 \\ 0 & -1 & 2 \end{vmatrix}}{1} = \frac{-8+3}{1} = \frac{-5}{1} = -5$$

$$z = \frac{\begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 0 & 3 & -1 \end{vmatrix}}{1} = \frac{-3+12-2}{1} = \frac{7}{1} = 7.$$

Solució $(-1, -5, 7)$

c) $\begin{cases} x + y + 2z = 2 \\ x \quad - z = 0 \\ \quad y - z = -1 \end{cases}$ $\quad A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

Es compleixen les condicions de Cràmer, ja que tenim que el nº eq = nº incògnites i $|A| \neq 0$:

$|A| = 2 + 1 + 1 = 4$

$$x = \frac{\begin{vmatrix} 2 & 1 & 2 \\ 0 & 0 & -1 \\ -1 & 1 & -1 \end{vmatrix}}{4} = \frac{1+2}{4} = \frac{3}{4}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 2 \\ 1 & 0 & -1 \\ 0 & -1 & -1 \end{vmatrix}}{4} = \frac{-2+2-1}{4} = \frac{-1}{4}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}}{4} = \frac{2+1}{4} = \frac{3}{4}.$$

Solucions $x = \frac{3}{4},\ y = -\frac{1}{4},\ z = \frac{3}{4}$

d)
$$\begin{cases} 3x - 2y = 4 \\ y - z = 4 \\ 2x + 2z = 4 \end{cases} \qquad A = \begin{pmatrix} 3 & -2 & 0 \\ 0 & 1 & -1 \\ 2 & 0 & 2 \end{pmatrix}$$

Es compleixen les condicions de Cramer: 3 incòg. i 3 eq., $|A| = 6 + 4 = 10$

$$x = \frac{\begin{vmatrix} 4 & -2 & 0 \\ 4 & 1 & -1 \\ 4 & 0 & 2 \end{vmatrix}}{10} = \frac{8+8+16}{10} = \frac{32}{10} = \frac{16}{5}$$

$$y = \frac{\begin{vmatrix} 3 & 4 & 0 \\ 0 & 4 & -1 \\ 2 & 4 & 2 \end{vmatrix}}{10} = \frac{24-8+12}{10} = \frac{28}{10} = \frac{14}{5}$$

$$z = \frac{\begin{vmatrix} 3 & -2 & 4 \\ 0 & 1 & 4 \\ 2 & 0 & 4 \end{vmatrix}}{10} = \frac{12-16-8}{10} = \frac{-12}{10} = \frac{-6}{2}$$

Sol: $\left(\frac{16}{5}, \frac{14}{5}, -\frac{6}{2}\right)$

e)
$$\begin{cases} 2x + 3y + 4z = 0 \\ -5x - 4y - 3z = 0 \\ x + y + 2z = 0 \end{cases} \qquad A = \begin{pmatrix} 2 & 3 & 4 \\ -5 & -4 & -3 \\ 1 & 1 & 2 \end{pmatrix}$$

$|A| = -16 - 9 - 20 + 16 + 30 + 6 = 7 \neq 0$

Es compleixen les condicions de la regla de Cramer.

$$x = \frac{\begin{vmatrix} 0 & 3 & 4 \\ 0 & -4 & -3 \\ 0 & 1 & 2 \end{vmatrix}}{7} = \frac{0}{7} = 0$$

$$y = \frac{\begin{vmatrix} 2 & 0 & 4 \\ -5 & 0 & -3 \\ 1 & 0 & 2 \end{vmatrix}}{7} = \frac{0}{7} = 0$$

$$z = \frac{\begin{vmatrix} 2 & 3 & 0 \\ -5 & -4 & 0 \\ 1 & 1 & 0 \end{vmatrix}}{7} = \frac{0}{7} = 0.$$

Sol. $(0,0,0)$

<u>NOTA</u>: Quan el $\text{rg}A = 3 \Rightarrow$ els sistemes homogenis són compatible determinats i tenen com a solució sempre la trivial: $(0,0,0)$

f) $\begin{cases} x + 2y + 3z = 1 \\ 2x - y + z = 1 \\ x + y + z = 0 \end{cases}$ $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$|A| = -1 + 2 + 6 + 3 - 4 - 1 = 5 \neq 0$

Es compleixen les condicions de la regla de Cràmer.

$$x = \frac{\begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & 1 \\ 0 & 1 & 1 \end{vmatrix}}{5} = \frac{-1+3-2-1}{5} = \boxed{\frac{-1}{5}}$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}}{5} = \frac{1+1-3-2}{5} = \boxed{\frac{-3}{5}}$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & 0 \end{vmatrix}}{5} = \frac{2+2+1-1}{5} = \boxed{\frac{4}{5}}$$

UFB

Sistemes d'equacions

107) Classifiqueu els sistemes d'equacions següents:

a) $\begin{cases} 3x - y = 2 \\ 2x + y + z = 0 \\ 6x - 2y = -1 \end{cases}$

Siguin

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 6 & -2 & 0 \end{pmatrix}, \quad M = \begin{pmatrix} 3 & -1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 6 & -2 & 0 & -1 \end{pmatrix}$$

la matriu de coeficients i ampliada, respectivament. Hem d'estudiar el rg A i rg M.

$|A| = -6 + 6 = 0 \Rightarrow rg\, A \neq 3.$

Però $\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 3 + 2 = 5 \neq 0 \Rightarrow rg\, A = 2.$

Hem de veure si rg M és 2 o no.

$$A_1 = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 6 & -2 & -1 \end{vmatrix} = -3 - 8 - 12 - 2 = -25 \neq 0$$

$\Rightarrow rg\, M = 3 \Rightarrow$ el sistema és <u>incompatible</u>.

b) $\begin{cases} 3x - y = 2 \\ 2x + y + z = 0 \\ 5x + z = 2 \end{cases}$

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 3 & -1 & 0 & 2 \\ 2 & 1 & 1 & 0 \\ 5 & 0 & 1 & 2 \end{pmatrix}$$

$|A| = 3 - 5 + 2 = 0 \Rightarrow rg\, A \neq 3.$

$$\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 3 + 2 = 5 \neq 0 \Rightarrow rg\, A = 2$$

$$\Delta_1 = \begin{vmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \\ 5 & 0 & 2 \end{vmatrix} = 6 - 10 + 4 = 0$$

$$\Delta_2 = \begin{vmatrix} 3 & 0 & 2 \\ 2 & 1 & 0 \\ 5 & 1 & 2 \end{vmatrix} = 6 + 4 - 10 = 0$$

$$\Delta_3 = \begin{vmatrix} -1 & 0 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = -2 + 2 = 0$$

$\Rightarrow rg\, M \neq 3$ i $rg\, M = 2$ perquè hi ha el matrix menor.

$\Rightarrow$ C.I.

c) $$\begin{cases} x + y - z + t = 1 \\ x - y - t = 2 \\ z - t = 0 \end{cases} \qquad A = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$M = \left(\begin{array}{cccc|c} 1 & 1 & -1 & 1 & 1 \\ 1 & -1 & 0 & -1 & 2 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right)$$

$rg\, A \leq 3$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 - 1 = -2 \neq 0 \Rightarrow rg\, A = 3$$

$\Rightarrow rg\, M = 3.$

Com que hi ha 4 incògnites $\Rightarrow$ C.I.

d) $\begin{cases} x - y - z + t = 4 \\ x + y + z - t = 2 \end{cases}$

$$A = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & -1 & -1 & 1 & 4 \\ 1 & 1 & 1 & -1 & 2 \end{pmatrix}$$

rg $A, M \leq 2$.

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 + 1 = 2 \neq 0 \Rightarrow \text{rg } A = 2 \Rightarrow \text{rg } M = 2$$

$\Rightarrow$ C.I.

e) $\begin{cases} 3x - y = 0 \\ 2x + y + z = 0 \\ 3x - 2y - z = 0 \end{cases}$ $\quad A = \begin{pmatrix} 3 & -1 & 0 \\ 2 & 1 & 1 \\ 3 & -2 & -1 \end{pmatrix}$

$$M = \begin{pmatrix} 3 & -1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & -2 & -1 & 0 \end{pmatrix}$$

$|A| = -3 - 3 - 2 + 6 = -2 \neq 0 \Rightarrow \text{rg } A = 3$

$\Rightarrow \text{rg } M = 3 \Rightarrow$ C.D.

108) Discutiu els sistemes següents segons els valors del paràmetre m:

a) $\begin{cases} mx + y + z = 4 \\ x + y + z = m \\ x - y + mz = 2 \end{cases}$ $\quad A = \begin{pmatrix} m & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & m \end{pmatrix}$

$$M = \begin{pmatrix} m & 1 & 1 & 4 \\ 1 & 1 & 1 & m \\ 1 & -1 & m & 2 \end{pmatrix}$$

$|A| = m^2 + \cancel{1} - \cancel{1} - 1 - \cancel{m} + \cancel{m} = m^2 - 1$

$m^2 - 1 = 0 \Rightarrow m = \pm 1$

- Si $m \neq 1, m \neq -1 \Rightarrow |A| \neq 0 \Rightarrow rg\,A = 3$

$\Rightarrow rg\,M = 3 \Rightarrow$ C.D.

- Si $m = 1 \Rightarrow |A| = 0 \Rightarrow rg\,A \neq 3$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \quad M = \begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 \neq 0 \Rightarrow rg\,A = 2$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 4 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 2 + 1 - 4 - 4 - 2 + 1$$
$$= -6 \neq 0 \Rightarrow rg\,M = 3$$

$\Rightarrow$ Incompatible

- Si $m = -1 \Rightarrow |A| = 0 \Rightarrow rg\,A \neq 3$

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \quad M = \begin{pmatrix} -1 & 1 & 1 & 4 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2 \Rightarrow rg\,A = 2.$$

$$\Delta_1 = \begin{vmatrix} -1 & 1 & 4 \\ 1 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} = -2 - \cancel{1} - 4 - 4 - 2 + \cancel{1}$$
$$= -12 \neq 0$$

$\Rightarrow rg\,M = 3 \Rightarrow$ incompatible.

b) $\begin{cases} x + 2y + 3z = 0 \\ x + my + z = 0 \\ 2x + 3y + 4z = 2 \end{cases}$

$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & m & 1 \\ 2 & 3 & 4 \end{pmatrix}$ $\quad M = \begin{pmatrix} 1 & 2 & 3 & 3 \\ 1 & m & 1 & 1 \\ 2 & 3 & 4 & 4 \end{pmatrix}$

$|A| = 4m + 4 + 9 - 6m - 8 - 3$

$= -2m + 2$

$-2m + 2 = 0 \rightarrow m = \frac{-2}{-2} = 1$

- Si $m \neq 1 \Rightarrow |A| \neq 0 \Rightarrow rg\, A = 3 \Rightarrow rg\, M = 3$

$\Rightarrow$ C.D

- Si $m = 1 \Rightarrow |A| = 0 \Rightarrow rg\, A \neq 3$

$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$ $\quad \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1 \neq 0$

$\Rightarrow rg\, A = 2$

$M = \begin{pmatrix} 1 & 2 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

c) $\begin{cases} x + my + z = 4 \\ x + 3y + z = 5 \\ mx + y + z = 4 \end{cases}$

$$A = \begin{pmatrix} 1 & m & 1 \\ 1 & 3 & 1 \\ m & 1 & 1 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & m & 1 & 4 \\ 1 & 3 & 1 & 5 \\ m & 1 & 1 & 4 \end{pmatrix}$$

$$|A| = 3 + m^2 + 1 - 3m - m - 1 = m^2 - 4m + 3$$

$$m^2 - 4m + 3 = 0 \Rightarrow m = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm 2}{2} \begin{cases} 3 \\ 1 \end{cases}$$

• Si $m \neq 1, m \neq 3 \Rightarrow |A| \neq 0 \Rightarrow rg\, A = 3 \Rightarrow rg\, M = 3$
$\Rightarrow$ C.D.

• Si $m = 1 \Rightarrow rg\, A \neq 3$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3 - 1 = 2 \neq 0$$

$\Rightarrow rg\, A = 2.$

$$M = \begin{pmatrix} 1 & 1 & 1 & 4 \\ 1 & 3 & 1 & 5 \\ 1 & 1 & 1 & 4 \end{pmatrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 4 \\ 1 & 3 & 5 \\ 1 & 1 & 4 \end{vmatrix} = 12 + 5 + 4 - 12 - 4 - 5 = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 4 \\ 1 & 1 & 5 \\ 1 & 1 & 4 \end{vmatrix} = 0 \quad \text{(2 columnas iguales)}$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 4 \\ 3 & 1 & 5 \\ 1 & 1 & 4 \end{vmatrix} = 4 + 5 + 12 - 4 - 12 - 5 = 0$$

$\Rightarrow rg\, M = 2 \Rightarrow$ C.I.

• Si $m = 3 \Rightarrow |A| = 0 \Rightarrow rg\,A \neq 3$

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \end{pmatrix} \qquad \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1 - 9 = -8 \neq 0$$

$\Rightarrow rg\,A = 2$

$$M = \begin{pmatrix} 1 & 3 & 1 & 4 \\ 1 & 3 & 1 & 5 \\ 3 & 1 & 1 & 4 \end{pmatrix}$$

$$\Delta_1 = \begin{vmatrix} 1 & 3 & 4 \\ 1 & 3 & 5 \\ 3 & 1 & 4 \end{vmatrix} = 12 + 45 + 4 - 36 - 12 - 5 = 8 \neq 0$$

$\Rightarrow rg\,M = 3 \Rightarrow$ incompatible

d) $\begin{cases} mx + y + z = m \\ x + y + z = 3 \\ x + y + mz = 3 \end{cases}$ $\qquad A = \begin{pmatrix} m & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & m \end{pmatrix}$

$$M = \begin{pmatrix} m & 1 & 1 & m \\ 1 & 1 & 1 & 3 \\ 1 & 1 & m & 3 \end{pmatrix}$$

$|A| = m^2 + 1 + 1 - 1 - m - m = m^2 + 1 - 2m$

$m^2 - 2m + 1 = 0 \Rightarrow m = \dfrac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$= \dfrac{2 \pm 0}{2} = 1.$

• Si $m \neq 1 \Rightarrow |A| \neq 0 \Rightarrow rg\,A = 3 = rg\,M \Rightarrow$ C.D.

• Si $m = 1 \Rightarrow |A| = 0 \Rightarrow rg\,A \neq 3$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Com que tots els elements son iguals, tots els menors d'ordre 2 son nuls $\Rightarrow$ rg $A \neq 2$

Però $|1| \neq 0 \Rightarrow$ rg $A = 1$.

D'altra banda,

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & 3 \end{pmatrix}$$

Tots els menors d'ordre 3 son nuls, ja que tenen dues columnes iguals

Per tant, rg $M \neq 3$.

I tenim que $\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3 - 1 = 2 \neq 0$

$\Rightarrow$ rg $M = 2 \Rightarrow$ incompatible.

109) Resol, si es pot, els sistemes següents:

a) $\begin{cases} -x + 2y + z = 3 \\ 5x - y + 4z = 3 \\ -3x + 3y - 5z = -2 \end{cases}$

En primer lloc, vegem si el sistema té o no solució.

$$A = \begin{pmatrix} -1 & 2 & 1 \\ 5 & -1 & 4 \\ -3 & 3 & -5 \end{pmatrix} \qquad M = \begin{pmatrix} -1 & 2 & 1 & 3 \\ 5 & -1 & 4 & 3 \\ -3 & 3 & -5 & -5 \end{pmatrix}$$

$|A| = -5 - 24 + 15 - 3 + 50 + 12 = 45 \Rightarrow$ rg $A = 3$

$\Rightarrow$ rg $M = 3 \Rightarrow$ CD.

Per tant té solució i es única. Aplicarem la regla de Crámer.

$$x = \frac{\begin{vmatrix} 3 & 2 & 1 \\ 3 & -1 & 4 \\ -2 & 3 & -5 \end{vmatrix}}{45} = \frac{15-16+9-2+30-36}{45} = 0$$

$$y = \frac{\begin{vmatrix} -1 & 3 & 1 \\ 5 & 3 & 4 \\ -3 & -2 & -5 \end{vmatrix}}{45} = \frac{15-36-10+9+75-8}{45} = 1.$$

$$z = \frac{\begin{vmatrix} -1 & 2 & 3 \\ 5 & -1 & 3 \\ -3 & 3 & -2 \end{vmatrix}}{45} = \frac{-2-18+45-9+20+9}{45} = 1.$$

Soluçió $x=0,\ y=1,\ z=1$

b) $\begin{cases} 2x+2y+5z=1 \\ x-y+3z=-4 \\ 3x-4y+z=-6 \end{cases}$ Mirem primer si té solució

$$A = \begin{pmatrix} 2 & 2 & 5 \\ 1 & -1 & 3 \\ 3 & -4 & 1 \end{pmatrix} \qquad M = \begin{pmatrix} 2 & 2 & 5 & 1 \\ 1 & -1 & 3 & -4 \\ 3 & -4 & 1 & -6 \end{pmatrix}$$

$$|A| = -2+18-20+15-2+24 = 33 \neq 0$$

$$\Rightarrow rg A = 3 = rg M \Rightarrow C.D.$$

Podem aplicar doncs la regla de Crámer per trobar la solució.

$$x = \frac{\begin{vmatrix} 1 & 2 & 5 \\ -4 & -1 & 3 \\ -6 & -4 & 1 \end{vmatrix}}{33} = \frac{-1-36+80-30+8+12}{33} = \frac{33}{33} = 1$$

$$y = \frac{\begin{vmatrix} 2 & 1 & 5 \\ 1 & -4 & 3 \\ 3 & -6 & 1 \end{vmatrix}}{33} = \frac{-8+9-30+60-1+36}{33} = 2.$$

$$z = \frac{\begin{vmatrix} 2 & 2 & 1 \\ 1 & -1 & -4 \\ 3 & -4 & -6 \end{vmatrix}}{33} = \frac{12-24-4+3+12-32}{33} = -1.$$

Solució $x=1$, $y=2$, $z=-1$.

c) $\begin{cases} x-3y+8z=2 \\ x+3y-z=8 \\ -x+2y+z=-3 \end{cases}$

$$A = \begin{pmatrix} 1 & -3 & 8 \\ 1 & 3 & -1 \\ -1 & 2 & 1 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & -3 & 8 & 2 \\ 1 & 3 & -1 & 8 \\ -1 & 2 & 1 & -3 \end{pmatrix}$$

$$|A| = 3-3+16+24+3+2 = 45 \neq 0$$

$$\Rightarrow \left.\begin{matrix} \text{rg } A = 3 = \text{rg } M \\ 3 \text{ incògnites} \end{matrix}\right\} \Rightarrow \text{C.D.}$$

Fem Cràmer.

$$x = \frac{\begin{vmatrix} 2 & -3 & 8 \\ 8 & 3 & -1 \\ -3 & 2 & 1 \end{vmatrix}}{45} = \frac{6-9+128+72+24+4}{45} = \frac{225}{45} = 5$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 8 \\ 1 & 8 & -1 \\ -1 & -3 & 1 \end{vmatrix}}{45} = \frac{8+2-24+64-2-3}{45} = \frac{45}{45} = 1$$

$$z = \frac{\begin{vmatrix} 1 & -3 & 8 \\ 1 & 3 & -1 \\ -1 & 2 & 1 \end{vmatrix}}{45} = \frac{3-3+16+24+3+2}{45} = 1$$

d) $\begin{cases} x + 2y + z = 1 \\ 6x - 4y + 7z = 11 \\ -x + 2y + 3z = 2 \end{cases}$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 6 & -4 & 7 \\ -1 & 2 & 3 \end{pmatrix} \qquad M = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 6 & -4 & 7 & 11 \\ -1 & 2 & 3 & 2 \end{pmatrix}$$

- $|A| = \cancel{-12} - 14 \cancel{+12} - 4 - 36 - 14$
  $= -68 \neq 0 \Rightarrow rg\, A = 3 \Rightarrow rg\, M = 3 \Rightarrow$ C.D.

- Apliquem la regla de Cramer.

$$x = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 11 & -4 & 7 \\ 2 & 2 & 3 \end{vmatrix}}{-68} = \frac{-12+28+22+8-66-14}{-68} = \frac{-34}{-68} = \frac{1}{2}$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 6 & 11 & 7 \\ -1 & 2 & 3 \end{vmatrix}}{-68} = \frac{33-7+12+11-18-14}{-68} = -\frac{17}{68}$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 6 & -4 & 11 \\ -1 & 2 & 2 \end{vmatrix}}{-68} = \frac{-8-22+12-4-24-22}{-68} = \frac{-68}{-68} = 1.$$

e) $\begin{cases} x + y - 2z = 0 \\ -x + y - z = 0 \\ -2x + 4y - 5z = 0 \end{cases}$ $\qquad A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 1 & -1 \\ -2 & 4 & -5 \end{pmatrix}$

$$M = \begin{pmatrix} 1 & 1 & -2 & 0 \\ -1 & 1 & -1 & 0 \\ -2 & 4 & -5 & 0 \end{pmatrix}$$

$|A| = -5 + 2 + 8 - 4 - 5 + 4 = 0 \Rightarrow rg\, A \neq 3$

$\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 1 + 1 = 2 \neq 0 \Rightarrow rg\, A = 2.$

Com que ampliem M amb una ~~fila~~ columna de zeros, llavors tots els menors d'ordre 3 de M són nuls.

$\Rightarrow \text{rg}\, M = 2 \Rightarrow$ C.I.

Hem de parametritzar una variable i dir les equacions independents

Com que $\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \neq 0 \Rightarrow$ les dues primeres equacions són independents. Per tant el sistema queda.

$$\begin{cases} x + y - 2z = 0 \\ -x + y - z = 0 \end{cases}$$

Sigui $z = \lambda$ un nombre qualsevol.

El sistema queda:

$$\begin{cases} x + y = 2\lambda \\ -x + y = \lambda \end{cases}$$

$$2y = 3\lambda$$

$$y = \frac{3\lambda}{2}$$

$$x = 2\lambda - \frac{3\lambda}{2} = \frac{4\lambda - 3\lambda}{2} = \frac{\lambda}{2}$$

$\Rightarrow$ les solucions del sistema són

$$x = \frac{\lambda}{2}, \quad y = \frac{3\lambda}{2}, \quad z = \lambda$$