

VFB.

## MATRICES

77) Calculeu el valor de  $x$  perquè les matrius  $A$  i  $B$  siguin iguals, amb

$$A = \begin{pmatrix} 3 & -x+4 \\ -2 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 0 \\ -2 & 0 \end{pmatrix}$$

$$\text{Si } A=B \Rightarrow \begin{cases} 3=3 \\ -2=-2 \\ -x+4=0 \\ 0=0 \end{cases} \Rightarrow \begin{cases} -x=-4 \\ \boxed{x=4} \end{cases}$$

(simultàniament)

78) Calculeu  $A-B$ ,  $B-A$  i  $-A+B$  amb

$$A = \begin{pmatrix} -1 & 0 & 3 \\ 0 & -4 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & -2 & -3 \\ 5 & 4 & -5 \end{pmatrix}$$

$$\begin{aligned} A-B &= \begin{pmatrix} -1+1 & 0+2 & 3+3 \\ 0-5 & -4-4 & 2+5 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 2 & 6 \\ -5 & -8 & 7 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} B-A &= \begin{pmatrix} -1+1 & -2-0 & -3-3 \\ 5-0 & 4+4 & -5-2 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -2 & -6 \\ 5 & 8 & -7 \end{pmatrix} \end{aligned}$$

Noteu que  $A-B = -(B-A)$ , ja que tenen signes oposats. És a dir  $A-B$  i  $B-A$  són matrius oposades.

$$-A+B = \begin{pmatrix} 1-1 & 0-2 & -3-3 \\ 0+7 & 4+4 & -2-5 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2 & -6 \\ 7 & 8 & -7 \end{pmatrix}$$

79) Calculez

$$-3 \cdot \begin{pmatrix} -1 & 0 & 3 \\ 0 & -4 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 0 & -9 \\ 0 & 12 & -6 \end{pmatrix}$$

$$5 \cdot \begin{pmatrix} 1 & -3 & 4 \\ 0 & 5 & 2 \\ 2 & -5 & -1 \end{pmatrix} = \begin{pmatrix} 5 & -15 & 20 \\ 0 & 25 & 10 \\ 10 & -25 & -5 \end{pmatrix}$$

80) Ecrivez les matrices transposées de

$$A = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 15 & 6 \\ 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 4 \\ -5 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 15 & 2 \\ -1 & 6 & 3 \end{pmatrix}, \quad B^t = \begin{pmatrix} -1 & -5 & 1 & 2 \\ 4 & 1 & 1 & 1 \end{pmatrix}$$

81) Calculez les produits de les matrices suivants:

$$a) \begin{pmatrix} -2 & -1 & 0 \\ 3 & 5 & 2 \\ -4 & 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} -1 & 2 \\ 0 & -5 \\ 3 & 2 \end{pmatrix}$$

$3 \times 3$                        $3 \times 2$

Les pouvons multiplier i us  
donc une matrice d'ordre  $3 \times 2$

$$\begin{pmatrix} -2 \cdot (-1) + 0 + 0 & -2 \cdot 2 + (-1) \cdot (-1) + 0 \\ 3 \cdot (-1) + 0 + 2 \cdot 3 & 3 \cdot 2 + 5 \cdot (-1) + 2 \cdot 2 \\ -4 \cdot (-1) + 0 + 6 \cdot 3 & -4 \cdot 2 + 0 + 6 \cdot 2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 3 & -15 \\ 22 & 4 \end{pmatrix}$$

$$b) \begin{pmatrix} 1 & 2 \\ 0 & 8 \end{pmatrix}_{2 \times 2} \cdot \begin{pmatrix} 4 & 2 & 1 \\ 0 & -3 & -2 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 4+0 & 2-6 & 1-4 \\ 0+0 & 0-24 & 0-16 \end{pmatrix}_{2 \times 3}$$

$$= \begin{pmatrix} 4 & -4 & -3 \\ 0 & -24 & -16 \end{pmatrix}$$

$$c) \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}_{2 \times 2} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 8 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 3 & 6 \\ 0 & -16 \end{pmatrix}_{2 \times 2}$$

82) Calculați și arătați că matricea inversă de la  
matricea

$$A = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 3 & -2 \\ 0 & 5 & -1 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & -3 \\ -1 & 3 & -2 \\ 0 & 5 & -1 \end{vmatrix} = -3 + 15 + 10 = 22 \neq 0$$

$\Rightarrow$  Cum știm  $|A| \neq 0$ ,  $A$  este inversabilă.

Amem a calcula:

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 3 & -2 \\ 5 & -1 \end{vmatrix} & -\begin{vmatrix} -1 & -2 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 0 & 5 \end{vmatrix} \\ -\begin{vmatrix} 0 & -3 \\ 5 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -3 \\ 0 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 5 \end{vmatrix} \\ \begin{vmatrix} 0 & -3 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & -3 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ -1 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -1 & -5 \\ -15 & -1 & -5 \\ 9 & 5 & 3 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{Adj}(A)^t}{|A|} = \frac{\begin{pmatrix} 7 & -1 & -5 \\ -15 & -1 & -5 \\ 9 & 5 & 3 \end{pmatrix}^t}{22}$$

$$= \frac{\begin{pmatrix} 7 & -15 & 9 \\ -1 & -1 & 5 \\ -5 & -5 & 3 \end{pmatrix}}{22} = \begin{pmatrix} \frac{7}{22} & \frac{-15}{22} & \frac{9}{22} \\ \frac{-1}{22} & \frac{-1}{22} & \frac{5}{22} \\ \frac{-5}{22} & \frac{-5}{22} & \frac{3}{22} \end{pmatrix}$$

83) Calculez la matrice inverse de B en fonction du paramètre  $\alpha$ , avec

$$B = \begin{pmatrix} 2 & -7 & \alpha \\ 1 & 3 & 3 \\ 0 & 1 & -4 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 2 & -7 & \alpha \\ 1 & 3 & 3 \\ 0 & 1 & -4 \end{vmatrix} = -24 + \alpha - 28 - 6 = \alpha - 58.$$

$$\alpha - 58 = 0 \Rightarrow \alpha = 58.$$

• Si  $\alpha = 58 \Rightarrow |B| = 0 \Rightarrow B$  n'est pas inversible.

• Si  $\alpha \neq 58 \Rightarrow |B| \neq 0 \Rightarrow B$  est inversible.

$$\Rightarrow \text{Adj}(B) = \begin{pmatrix} \begin{vmatrix} 3 & 3 \\ 1 & -4 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 0 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} -7 & \alpha \\ 1 & -4 \end{vmatrix} & \begin{vmatrix} 2 & \alpha \\ 0 & -4 \end{vmatrix} & -\begin{vmatrix} 2 & -7 \\ 0 & 1 \end{vmatrix} \\ \begin{vmatrix} -7 & \alpha \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & \alpha \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -7 \\ 1 & 3 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -15 & 4 & 1 \\ -28+\alpha & -8 & -2 \\ -21-3\alpha & -6+\alpha & 13 \end{pmatrix}$$

$$\Rightarrow B^{-1} = \begin{pmatrix} -15 & -28+\alpha & -21-3\alpha \\ 4 & -8 & -6+\alpha \\ 1 & -2 & 13 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-15}{\alpha-58} & \frac{\alpha-28}{\alpha-58} & \frac{-3\alpha-21}{\alpha-58} \\ \frac{4}{\alpha-58} & \frac{-8}{\alpha-58} & \frac{\alpha-6}{\alpha-58} \\ \frac{1}{\alpha-58} & \frac{-2}{\alpha-58} & \frac{13}{\alpha-58} \end{pmatrix}$$

84) Calculeu el valor del rang de la matriu:

$$\begin{pmatrix} 0 & 0 & 3 & -2 \\ -4 & 2 & 3 & 8 \end{pmatrix}$$

• Sabem que el rang d'aquesta matriu és  $\leq \min\{2, 4\} = 2$ .

• Anem a veure si pot ser 2. Prenem un determinant quadrat d'ordre 2:

$$\begin{vmatrix} 3 & -2 \\ 3 & 8 \end{vmatrix} = 24 + 6 = 30 \neq 0.$$

Com que aquest menor és diferent de 0, llavors ~~serà~~ el rang és 2.

85) Calculeu  $\text{rg } A$  en funció del paràmetre  $\alpha$ , on

$$A = \begin{pmatrix} 2 & -7 & \alpha \\ 1 & 3 & 3 \\ 0 & 1 & -4 \end{pmatrix}$$

Sabem que  $\text{rg } A \leq 3$ . Vegem si pot ser 3.

$$|A| = \begin{vmatrix} 2 & -7 & \alpha \\ 1 & 3 & 3 \\ 0 & 1 & -4 \end{vmatrix} = -24 + \alpha - 28 - 6 \\ = \alpha - 58.$$

$$\alpha - 58 = 0 \Rightarrow \alpha = 58.$$

• Si  $\alpha \neq 58 \Rightarrow |A| \neq 0 \Rightarrow \text{rg } A = 3$

• Si  $\alpha = 58 \Rightarrow |A| = 0$  i  $A = \begin{pmatrix} 2 & -7 & 58 \\ 1 & 3 & 3 \\ 0 & 1 & -4 \end{pmatrix}$

i  $\text{rg } A \neq 3$ . Mirarem si  $\text{rg } A$  és 2.

Per això hem de trobar un menor d'ordre 2 que no sigui nul.

$$\Delta = \begin{vmatrix} 2 & -7 \\ 1 & 3 \end{vmatrix} = 6 + 7 = 13 \neq 0$$

Com que  $\Delta \neq 0 \Rightarrow \text{rg } A = 2$ .

En conclusió, si  $\alpha \neq 58 \Rightarrow \text{rg } A = 3$  i si

$$\alpha = 58 \Rightarrow \text{rg } A = 2.$$



86) Donades les matrius

$$A = \begin{pmatrix} 2 & 0 & -3 \\ -2 & 1 & 0 \\ 2 & 1 & 3 \end{pmatrix} \quad ; \quad B = \begin{pmatrix} 2 & 0 \\ -2 & 1 \\ 2 & 1 \end{pmatrix}$$

calcular, si és possible,  $AB$  i  $BA$ .

$$\begin{array}{l} A \text{ té ordre } 3 \times 3 \\ B \text{ té ordre } 3 \times 2 \end{array}$$

$\Rightarrow$   $AB$  es pot calcular  
i té ordre  $3 \times 2$

$BA$  no es pot calcular.

$$A \cdot B = \begin{pmatrix} -2 & -3 \\ -6 & 1 \\ 8 & 4 \end{pmatrix}$$

87) Calcular  $3AA^t - 2I$ , amb  $A = \begin{pmatrix} 2 & 0 & -3 \\ -2 & 1 & 0 \\ 2 & 1 & 3 \end{pmatrix}$

$$3AA^t - 2I = 3 \begin{pmatrix} 2 & 0 & -3 \\ -2 & 1 & 0 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 & -2 & 2 \\ 0 & 1 & 1 \\ -3 & 0 & 3 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 0 & -9 \\ -6 & 3 & 0 \\ 6 & 3 & 9 \end{pmatrix} \cdot \begin{pmatrix} 2 & -2 & 2 \\ 0 & 1 & 1 \\ -3 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 39 & -12 & -15 \\ -12 & 15 & -9 \\ -15 & -9 & 42 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 37 & -12 & -15 \\ -12 & 13 & -9 \\ -15 & -9 & 40 \end{pmatrix}$$

88) Comprova que  $(A \cdot B)^t = B^t \cdot A^t$  amb les matrius

$$A = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \quad ; \quad B = \begin{pmatrix} -1 & 0 \\ 3 & 6 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 14 & 30 \\ 10 & 24 \end{pmatrix}$$

$$\Rightarrow (A \cdot B)^t = \begin{pmatrix} 14 & 10 \\ 30 & 24 \end{pmatrix}$$

$$A^t = \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} \quad , \quad B^t = \begin{pmatrix} -1 & 3 \\ 0 & 6 \end{pmatrix}$$

$$\Rightarrow B^t \cdot A^t = \begin{pmatrix} -1 & 3 \\ 0 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 14 & 10 \\ 30 & 24 \end{pmatrix}$$

Per tant, efectivament  $(A \cdot B)^t = B^t \cdot A^t$ .

89) Determina el valor de  $m$  per els quals es verifica que  $X^2 - \frac{5}{2}X + I = 0$ , amb  $X = \begin{pmatrix} m & 0 \\ 0 & 2 \end{pmatrix}$

Si  $X$  verifica l'equació matricial, llavors:

$$\begin{pmatrix} m & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} m & 0 \\ 0 & 2 \end{pmatrix} - \frac{5}{2} \begin{pmatrix} m & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} m^2 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} \frac{5}{2}m & 0 \\ 0 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} m^2 - \frac{5}{2}m + 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

La igualtat de matrius implica la igualtat d'element a element. Per tant

$$m^2 - \frac{5}{2}m + 1 = 0.$$

$$\Rightarrow 2m^2 - 5m + 2 = 0$$

$$m = \frac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} = \frac{5 \pm 3}{4} \begin{cases} 2 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$$

$\Rightarrow$  Els valors de  $m$  cercats són 2 i  $\frac{1}{2}$ .

90) Determinem  $a$  i  $b$  de forma que es verifiqui  $A^2 = A$   
amb  $A = \begin{pmatrix} 2 & -1 \\ a & b \end{pmatrix}$

Si  $A$  verifica l'equació matricial:

$$\begin{pmatrix} 2 & -1 \\ a & b \end{pmatrix} \cdot \begin{pmatrix} 2 & -1 \\ a & b \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ a & b \end{pmatrix}$$

$$\begin{pmatrix} 4 - a & -2 - b \\ 2a + ab & -a + b^2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ a & b \end{pmatrix}$$

Per tant,

$$\left. \begin{array}{l} 4 - a = 2 \\ -2 - b = -1 \\ 2a + ab = a \\ -a + b^2 = b \end{array} \right\} \begin{array}{l} a = 2 \\ b = -2 + 1 = -1 \end{array}$$

Calçò és un sistema d'equacions. S'han de verificar les igualtats simultàniament)

Heu de comprovar si es verifiquen les anteriors equacions:

$$2 \cdot 2 + 2 \cdot (-1) = 2 \quad ?$$

$$4 - 2 = 2 \quad \text{sí}$$

$$-2 + (-1)^2 = -1 \quad ?$$

$$-2 + 1 = -1 \quad \text{sí}$$

Per tant el sistema té solució

$$a = 2$$

$$b = -1.$$

Aleshones els valors constants són  $a=2$ ;  $b=-1$ .

NOTA Si  $A$  verifica que  $A^2=A$  es diu que la matriu és idempotent.

91) Trobareu totes les matrius  $X$  de la forma:

$$X = \begin{pmatrix} a & 1 & 0 \\ 0 & b & 1 \\ 0 & 0 & c \end{pmatrix} \text{ tal que}$$

$$X^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X \cdot X = \begin{pmatrix} a & 1 & 0 \\ 0 & b & 1 \\ 0 & 0 & c \end{pmatrix} \cdot \begin{pmatrix} a & 1 & 0 \\ 0 & b & 1 \\ 0 & 0 & c \end{pmatrix}$$
$$= \begin{pmatrix} a^2 & a+b & 1 \\ 0 & b^2 & b+c \\ 0 & 0 & c^2 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a^2 & a+b & 1 \\ 0 & b^2 & b+c \\ 0 & 0 & c^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{cases} a^2 = 1 \Rightarrow a = \pm 1 \\ a+b = 0 \\ b^2 = 1 \Rightarrow b = \pm 1 \\ b+c = 0 \\ c^2 = 1 \Rightarrow c = \pm 1 \end{cases}$$

Si  $a = +1 \Rightarrow a+b = 0$  implica que  $1+b = 0 \Rightarrow b = -1$   
i  $b+c = 0$  " "  $-1+c = 0 \Rightarrow c = 1$

Si  $a = -1 \Rightarrow -1+b = 0 \Rightarrow b = 1$   
 $1+c = 0 \Rightarrow c = -1$ .

Pu tant, tenir dues matrius:

$$X = \begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

que compleixin amb la condició donada.

92) Calculeu dos nombres reals  $m$  i  $n$  tals que

$$A + mA + nI = 0 \quad \text{si} \quad A = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} + \begin{pmatrix} 2m & m \\ 2m & 3m \end{pmatrix} + \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2m+n+2 & m+1 \\ 2m+2 & 3m+n+3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2m+n+2 = 0 & \rightarrow 2 \cdot (-1) + n + 2 = 0 \\ m+1 = 0 & \Rightarrow m = -1 \\ 2m+2 = 0 & \Rightarrow m = -1. \\ 3m+n+3 = 0 & \end{cases} \quad \begin{matrix} \downarrow \\ n = 0 \end{matrix}$$

$$\downarrow \\ 3 \cdot (-1) + 0 + 3 = 0 \quad \text{si.}$$

$$\Rightarrow \boxed{m = -1 \text{ i } n = 0}$$

93) Siguen A i B los matrices

$$A = \begin{pmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} a & b & 0 \\ c & c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Troben las condiciones que han de cumplir los coeficientes a, b i c para que se verifique que  $AB = BA$ .

$$A \cdot B = \begin{pmatrix} 5a+2c & 5b+2c & 0 \\ 2a+5c & 2b+5c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 5a+2b & 2a+5b & 0 \\ 5c+2c & 2c+5c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AB = BA \Rightarrow \begin{cases} 5a+2c = 5a+2b \\ 5b+2c = 5b+2a \\ 5c+2a = 5c+2c \\ 2b+5c = 2c+5c \end{cases}$$

$$\Rightarrow \begin{aligned} c &= b \\ c &= a & \Rightarrow a = b = c \\ a &= c \\ b &= c. \end{aligned}$$

Por tanto,  $a = b = c = \lambda$  un número cualquiera.

94) Trebeu dues matrius  $X$  i  $Y$  que verifiquin el sistema següent:

$$\left. \begin{aligned} 2X - 3Y &= \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \\ X - Y &= \begin{pmatrix} -1 & 0 \\ 3 & 6 \end{pmatrix} \end{aligned} \right\}$$

$$\text{Siguem } A = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \quad ; \quad B = \begin{pmatrix} -1 & 0 \\ 3 & 6 \end{pmatrix}$$

Podem veure així el sistema matricial com:

$$\left\{ \begin{aligned} 2X - 3Y &= A \\ X - Y &= B \end{aligned} \right.$$

De la segona equació, tenim que  $X = B + Y$ .  
Per tant, substituïm a la primera equació:

$$2(B + Y) - 3Y = A$$

$$2B + 2Y - 3Y = A$$

$$2B - 1Y = A$$

$$-1Y = A - 2B$$

$$Y = -A + 2B$$

$$\text{I, per tant, } X = B + Y = B - A + 2B$$

$$= -A + 3B.$$

$$\text{Per tant, } Y = \begin{pmatrix} -1 & -5 \\ -2 & -4 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 6 & 12 \end{pmatrix} = \begin{pmatrix} -3 & -5 \\ 4 & 8 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & -5 \\ -2 & -4 \end{pmatrix} + \begin{pmatrix} -3 & 0 \\ 9 & 18 \end{pmatrix} = \begin{pmatrix} -4 & -5 \\ 7 & 14 \end{pmatrix}$$

95) Calculez, si es possible, la matrice inverse de cada uno de los matrices siguientes:

$$a) A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$$

$$|A| = 6 - 6 = 0 \Rightarrow A \text{ no es inversa}$$

$$b) B = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$$

$$|B| = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3 - 6 = -3 \neq 0 \rightarrow A \text{ es}$$

inversa.

$$\text{Adj}(B) = \begin{pmatrix} |3| & -|2| \\ -|2| & |1| \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix}$$

$$\text{Por tanto, } B^{-1} = \frac{\begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix}^t}{-3} = \frac{\begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}}{-3}$$

$$= \begin{pmatrix} -1 & \frac{2}{3} \\ +1 & -\frac{1}{3} \end{pmatrix}$$

$$c) C = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -5 & -2 \\ 3 & 3 & 6 \end{pmatrix}$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -5 & -2 \\ 3 & 3 & 6 \end{vmatrix} = -30 - 12 + 27 + 45 - 36 + 6$$

$$= 0$$

$\Rightarrow C$  no es inversa



$$d) D = \begin{pmatrix} 1 & 2 & 4 \\ 3 & -5 & -4 \\ 3 & 3 & 1 \end{pmatrix}$$

$$|D| = \begin{vmatrix} 1 & 2 & 4 \\ 3 & -5 & -4 \\ 3 & 3 & 1 \end{vmatrix} = -5 - 24 + 36 + 60 - 6 + 12 \\ = 73 \neq 0$$

$\Rightarrow D^{-1}$  existit.

$$\text{Adj}(D) = \begin{pmatrix} \begin{vmatrix} -5 & -4 \\ 3 & 1 \end{vmatrix} & - \begin{vmatrix} 3 & -4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 3 & -5 \\ 3 & 3 \end{vmatrix} \\ - \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ 3 & 1 \end{vmatrix} & - \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ -5 & -4 \end{vmatrix} & - \begin{vmatrix} 1 & 4 \\ 3 & -4 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -15 & 24 \\ 10 & -11 & 3 \\ 12 & 16 & -11 \end{pmatrix}$$

$$\Rightarrow D^{-1} = \frac{\begin{pmatrix} 7 & 10 & 12 \\ -15 & -11 & 16 \\ 24 & 3 & -11 \end{pmatrix}}{73} = \begin{pmatrix} \frac{7}{73} & \frac{10}{73} & \frac{12}{73} \\ \frac{-15}{73} & \frac{-11}{73} & \frac{16}{73} \\ \frac{24}{73} & \frac{3}{73} & \frac{-11}{73} \end{pmatrix}$$

96) Calculez le module inverse de cada uno de los matrices siguientes:

$$a) A = \begin{pmatrix} -1 & 2 \\ 3 & a \end{pmatrix}$$

$$|A| = -a - 6$$

$$-a - 6 = 0 \Rightarrow a = -6.$$

• Si  $a = -6 \Rightarrow |A| = 0 \Rightarrow A$  no te inversa.

• Si  $a \neq -6 \Rightarrow |A| \neq 0 \Rightarrow A$  te inversa.

En aquest cas, la podem calcular

$$\text{Adj}(A) = \begin{pmatrix} a & -3 \\ -2 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{\begin{pmatrix} a & -2 \\ -3 & -1 \end{pmatrix}}{-a-6} = \begin{pmatrix} \frac{a}{-a-6} & \frac{-2}{-a-6} \\ \frac{-3}{-a-6} & \frac{-1}{-a-6} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{-a}{a+6} & \frac{2}{a+6} \\ \frac{3}{a+6} & \frac{1}{a+6} \end{pmatrix}$$

b)  $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -5 & -2 \\ 3 & b & 6 \end{pmatrix}$

$$|B| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -5 & -2 \\ 3 & b & 6 \end{vmatrix} = -30 - 12 + 9b + 45 - 36 + 2b$$
$$= 11b - 33$$

$$11b - 33 = 0 \Rightarrow b = 3.$$

• Si  $b = 3 \Rightarrow |B| = 0 \Rightarrow B$  no te inversa

• Si  $b \neq 3 \Rightarrow |B| \neq 0 \Rightarrow B$  te inversa.

$$\text{Adj}(B) = \begin{pmatrix} \begin{vmatrix} -5 & -2 \\ b & 6 \end{vmatrix} & -\begin{vmatrix} 3 & -2 \\ 3 & 6 \end{vmatrix} & \begin{vmatrix} 3 & -5 \\ 3 & b \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ b & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 3 & b \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ -5 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -30 + 2b & -24 & 3b + 15 \\ -12 + 3b & -3 & -b + 6 \\ 11 & 11 & -11 \end{pmatrix}$$

Pu tant,

$$B^{-1} = \frac{\begin{pmatrix} -30 + 2b & -12 + 3b & 11 \\ -24 & -3 & 11 \\ 3b + 15 & -b + 6 & -11 \end{pmatrix}}{11b - 33}$$

$$= \begin{pmatrix} \frac{-30 + 2b}{11b - 33} & \frac{-12 + 3b}{11b - 33} & \frac{11}{11b - 33} \\ \frac{-24}{11b - 33} & \frac{-3}{11b - 33} & \frac{11}{11b - 33} \\ \frac{3b + 15}{11b - 33} & \frac{-b + 6}{11b - 33} & \frac{-11}{11b - 33} \end{pmatrix}$$

Com que  $11b - 33 = 11(b - 3)$ , podem simplificar, opcionalment, algunes expressions.

$$= \begin{pmatrix} \frac{2(b - 15)}{11(b - 3)} & \frac{3(b - 4)}{11(b - 3)} & \frac{1}{b - 3} \\ \frac{-24}{11(b - 3)} & \frac{-3}{11(b - 3)} & \frac{1}{b - 3} \\ \frac{3(b + 5)}{11(b - 3)} & \frac{-b + 6}{11(b - 3)} & \frac{-1}{b - 3} \end{pmatrix}$$

97) Digueu en funció dels paràmetres corresponents quan les matrius quadrats són regulars. En cas de no-ho, trobeu la seva inversa.

$$2) A = \begin{pmatrix} \alpha+2 & 1 & 1 \\ 1 & \alpha+2 & 1 \\ 1 & 1 & \alpha+2 \end{pmatrix}$$

• Calculam primer  $|A|$

$$|A| = \begin{vmatrix} \alpha+2 & 1 & 1 \\ 1 & \alpha+2 & 1 \\ 1 & 1 & \alpha+2 \end{vmatrix} = (\alpha+2)^3 + 1 + 1 - (\alpha+2) - (\alpha+2) - (\alpha+2)$$

$$= \alpha^3 + 6\alpha^2 + 12\alpha + 8 + 2 - 3(\alpha+2)$$

$$= \alpha^3 + 6\alpha^2 + 12\alpha + 10 - 3\alpha - 6$$

$$= \alpha^3 + 6\alpha^2 + 9\alpha + 4$$

$$\begin{aligned} (\alpha+2)^3 &= 1\alpha^3 + 3\alpha^2 \cdot 2 + 3\alpha \cdot 2^2 + 1 \cdot 2^3 \\ &= 1\alpha^3 + 6\alpha^2 + 12\alpha + 8 \end{aligned}$$

els coeficients són la fila 4 ( $n=3$ ) del triangle de Tartaglià

$$\begin{array}{ccc} 1 & & \leftarrow n=0 \\ 1 & 1 & \leftarrow n=1 \\ 1 & 2 & 1 & \leftarrow n=2 \end{array}$$

$$\boxed{1 \quad 3 \quad 3 \quad 1} \leftarrow n=3$$

• Volem saber quan  $|A|=0$ . Trobem les arrels reals del polinomi en qüestions.

$$\text{Div}(4) = \{ \pm 1, \pm 2, \pm 4 \}$$

$$\begin{array}{c|cccc} & 1 & 6 & 9 & 4 \\ \hline 1 & 1 & 7 & 16 & 20 \end{array}$$

$\Rightarrow 1$  no es-valor

$$\begin{array}{c|cccc} & 1 & 6 & 9 & 4 \\ \hline -1 & -1 & -5 & -4 & 0 \end{array}$$

$\Rightarrow -1$  es-valor

i

$$x^3 + 6x^2 + 9x + 4$$

$$= (x+1)(x^2 + 5x + 4)$$

$$x^2 + 5x + 4 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{25 - 4 \cdot 1 \cdot 4}}{2 \cdot 1}$$

$$= \frac{-5 \pm 3}{2} \begin{array}{l} -1 \\ -4 \end{array}$$

$\Rightarrow |A| = 0$  si i nois si  $x = -1$  o  $x = -4$ .

A més, per Ruffini,  $|A| = (x+1)^2(x+4)$

• Si  $x \neq -1$  i  $x \neq -4 \Rightarrow A$  es regular.

Calcular en aquest cas la seva inversa

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} x+2 & 1 \\ 1 & x+2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & x+2 \end{vmatrix} & \begin{vmatrix} 1 & x+2 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & x+2 \end{vmatrix} & \begin{vmatrix} x+2 & 1 \\ 1 & x+2 \end{vmatrix} & -\begin{vmatrix} x+2 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ x+2 & 1 \end{vmatrix} & -\begin{vmatrix} x+2 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} x+2 & 1 \\ 1 & x+2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (\alpha+2)^2 & -(\alpha+1) & -\alpha-1 \\ -(\alpha+1) & (\alpha+2)^2 & -(\alpha+1) \\ -\alpha-1 & -(\alpha+1) & (\alpha+2)^2 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} & \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} & \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} \\ \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} & \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} & \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} \\ \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} & \frac{-(\alpha+1)}{(\alpha+1)^2(\alpha+4)} & \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} \end{pmatrix}$$

$$\begin{matrix} \alpha-1 \\ -(\alpha+1) \end{matrix}$$

$$= \begin{pmatrix} \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} & \frac{-1}{(\alpha+1)(\alpha+4)} & \frac{-1}{(\alpha+1)(\alpha+4)} \\ \frac{-1}{(\alpha+1)(\alpha+4)} & \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} & \frac{-1}{(\alpha+1)(\alpha+4)} \\ \frac{-1}{(\alpha+1)(\alpha+4)} & \frac{-1}{(\alpha+1)(\alpha+4)} & \frac{(\alpha+2)^2}{(\alpha+1)^2(\alpha+4)} \end{pmatrix}$$

$$b) A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & 2 \\ -1 & 1 & a \end{pmatrix}$$

$$|A| = a^2 - 2 + 1 + a - a - 2 = a^2 - 3$$

$$a^2 - 3 = 0 \Rightarrow a^2 = 3 \Rightarrow a = \pm\sqrt{3}$$

$$\bullet \text{ Si } a \neq \sqrt{3} ; a \neq -\sqrt{3} \Rightarrow |A| \neq 0$$

$$\Rightarrow A \text{ es regular.}$$

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} a & 2 \\ 1 & a \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ -1 & a \end{vmatrix} & \begin{vmatrix} 1 & a \\ -1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ -1 & a \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ a & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a^2 - 2 & -a - 2 & 1 + a \\ -a + 1 & a + 1 & -2 \\ 2 - a & -1 & a - 1 \end{pmatrix}$$

$$= \begin{pmatrix} a^2 - 2 & -a - 2 & a + 1 \\ -a + 1 & a + 1 & -2 \\ -a + 2 & -1 & a - 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{a^2 - 2}{a^2 - 3} & \frac{-a + 1}{a^2 - 3} & \frac{-a + 2}{a^2 - 3} \\ \frac{-a - 2}{a^2 - 3} & \frac{a + 1}{a^2 - 3} & \frac{-1}{a^2 - 3} \\ \frac{a + 1}{a^2 - 3} & \frac{-2}{a^2 - 3} & \frac{a - 1}{a^2 - 3} \end{pmatrix}$$

$$c) A = \begin{pmatrix} -1 & 2 & 4 \\ 3 & 2 & a \\ -5 & -6 & 2 \end{pmatrix}$$

$$\begin{aligned} |A| &= -4 - 10a - 72 + 40 - 12 - 6a \\ &= -16a - 48 = 0 \Rightarrow a = -3 \end{aligned}$$

• Si  $a \neq -3 \Rightarrow A$  es invertible.

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 2 & a \\ -6 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & a \\ -5 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ -5 & -6 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ -6 & 2 \end{vmatrix} & \begin{vmatrix} -1 & 4 \\ -5 & 2 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ -5 & -6 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 2 & a \end{vmatrix} & -\begin{vmatrix} -1 & 4 \\ 3 & a \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 3 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 4+6a & -6-5a & -8 \\ -28 & 18 & -16 \\ 2a-8 & a+12 & -8 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{4+6a}{-16a-48} & \frac{-28}{-16a-48} & \frac{2a-8}{-16a-48} \\ \frac{-6-5a}{-16a-48} & \frac{18}{-16a-48} & \frac{a+12}{-16a-48} \\ \frac{-8}{-16a-48} & \frac{-16}{-16a-48} & \frac{-8}{-16a-48} \end{pmatrix}$$

$$d) A = \begin{pmatrix} x & 2 & -1 \\ 3 & 2 & x+1 \\ 7 & 6 & 1 \end{pmatrix}$$

$$|A| = 2x + 14(x+1) - 18 + 14 - 6 - 6x(x+1)$$

$$= 2x + 14x + 14 - 18 + 14 - 6 - 6x^2 - 6x$$

$$= -6x^2 + 10x + 4.$$

$$-6x^2 + 10x + 4 = 0 \Rightarrow x = \frac{-10 \pm \sqrt{100 - 4 \cdot (-6) \cdot 4}}{2 \cdot (-6)}$$

$$= \frac{-10 \pm 14}{-12} \begin{cases} -\frac{4}{12} = -\frac{1}{3} \\ 2 \end{cases}$$

Si  $x \neq 2$  ;  $x \neq -\frac{1}{3} \Rightarrow A$  es regular.



$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 2 & \alpha+1 \\ 6 & 1 \end{vmatrix} & - \begin{vmatrix} 3 & \alpha+1 \\ 7 & 1 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 7 & 6 \end{vmatrix} \\ - \begin{vmatrix} 2 & -1 \\ 6 & 1 \end{vmatrix} & \begin{vmatrix} \alpha & -1 \\ 7 & 1 \end{vmatrix} & - \begin{vmatrix} \alpha & 2 \\ 7 & 6 \end{vmatrix} \\ \begin{vmatrix} 2 & -1 \\ 2 & \alpha+1 \end{vmatrix} & - \begin{vmatrix} \alpha & -1 \\ 3 & \alpha+1 \end{vmatrix} & \begin{vmatrix} \alpha & 2 \\ 3 & 2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -6\alpha - 4 & +7\alpha + 4 & 4 \\ -8 & \alpha + 7 & -6\alpha + 14 \\ 2\alpha + 4 & -\alpha(\alpha+1) - 3 & -2\alpha - 6 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-6\alpha - 4}{-6\alpha^2 + 10\alpha + 4} & \frac{-8}{-6\alpha^2 + 10\alpha + 4} & \frac{2\alpha + 4}{-6\alpha^2 + 10\alpha + 4} \\ \frac{7\alpha + 4}{-6\alpha^2 + 10\alpha + 4} & \frac{\alpha + 7}{-6\alpha^2 + 10\alpha + 4} & \frac{-\alpha(\alpha+1) - 3}{-6\alpha^2 + 10\alpha + 4} \\ \frac{4}{-6\alpha^2 + 10\alpha + 4} & \frac{-6\alpha + 14}{-6\alpha^2 + 10\alpha + 4} & \frac{-2\alpha - 6}{-6\alpha^2 + 10\alpha + 4} \end{pmatrix}$$

$$c) A = \begin{pmatrix} 1 & a & 1 \\ a-1 & -2 & -1 \\ 1 & a+1 & 1 \end{pmatrix}$$

$$\begin{aligned} |A| &= -2 - a + (a-1)(a+1) + 2 - a(a-1) \\ &\quad + a+1 \\ &= \cancel{-2} - \cancel{a} + \cancel{a^2} - \cancel{1} + \cancel{2} - \cancel{a^2} + \cancel{a} \\ &\quad + \cancel{a} + \cancel{1} = a \end{aligned}$$

Si  $a \neq 0 \rightarrow A$  es regular.

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} -2 & -1 \\ a+1 & 1 \end{vmatrix} & -\begin{vmatrix} a-1 & -1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} a-1 & -2 \\ 1 & a+1 \end{vmatrix} \\ -\begin{vmatrix} a & 1 \\ a+1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & a \\ 1 & a+1 \end{vmatrix} \\ \begin{vmatrix} a & 1 \\ -2 & -1 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ a-1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & a \\ a-1 & -2 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -2+a+1 & -a+1-1 & (a-1)(a+1)+2 \\ -a+a+1 & 0 & -a-1+a \\ -a+2 & 1+a-1 & -2-a(a-1) \end{pmatrix}$$

$$= \begin{pmatrix} a-1 & -a & a^2+1 \\ 1 & 0 & -1 \\ -a+2 & a & -a^2+a-2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{a-1}{a} & \frac{1}{a} & \frac{-a+2}{a} \\ \frac{-a}{a} & 0 & \frac{a}{a} \\ \frac{a^2+1}{a} & -\frac{1}{a} & \frac{-a^2+a-2}{a} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{a-1}{a} & \frac{1}{a} & \frac{-a+2}{a} \\ -1 & 0 & 1 \\ \frac{a^2+1}{a} & -\frac{1}{a} & \frac{-a^2+a-2}{a} \end{pmatrix}$$

$$f) A = \begin{pmatrix} m & 0 & 2 \\ m & m & 4 \\ 0 & m & 2 \end{pmatrix}$$

$|A| = 2m^2 + 2m^2 - 4m^2 = 0 \Rightarrow$  Independentment de  $m$ ,  $|A|$  sempre dona 0  $\Rightarrow$  A mai té inversa.

$\Rightarrow$  A mai és regular  $\Rightarrow$  No podem donar la seva inversa (perquè no en té).

$$g) A = \begin{pmatrix} 0 & 1 & 1 \\ m & 4 & 4 \\ m & 2 & 1 \end{pmatrix} \quad |A| = \cancel{4m} + 2m - \cancel{4m} - m = m$$

Si  $m \neq 0 \Rightarrow$  A es regular.

En aquest cas,

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 4 & 4 \\ 2 & 1 \end{vmatrix} & - \begin{vmatrix} m & 4 \\ m & 1 \end{vmatrix} & \begin{vmatrix} m & 4 \\ m & 2 \end{vmatrix} \\ - \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ m & 1 \end{vmatrix} & - \begin{vmatrix} 0 & 1 \\ m & 2 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 4 & 4 \end{vmatrix} & - \begin{vmatrix} 0 & 1 \\ m & 4 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ m & 4 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 3m & -2m \\ 1 & -m & m \\ 0 & m & -m \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{-4}{m} & \frac{1}{m} & 0 \\ 3 & -1 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$

$$h) A = \begin{pmatrix} 1 & 1 & 0 \\ a & 0 & 1 \\ a+1 & 1 & a \end{pmatrix}$$

$$|A| = a+1 - a^2 - 1 = -a^2 + a = 0$$

$$a(-1a+1) = 0$$

$$\begin{cases} a=0 \\ a=1 \end{cases}$$

Si  $a \neq 0$  i  $a \neq 1 \Rightarrow A$  és regular.

En aquestes cas, podem calcular  $A^{-1}$ :

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 0 & 1 \\ 1 & a \end{vmatrix} & -\begin{vmatrix} a & 1 \\ a+1 & a \end{vmatrix} & \begin{vmatrix} a & 0 \\ a+1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 0 \\ 1 & a \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ a+1 & a \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ a+1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ a & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ a & 0 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -a^2+a+1 & a \\ -a & a & a \\ 1 & -1 & -a \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{-1}{-a^2+a} & \frac{-a}{-a^2+a} & \frac{1}{-a^2+a} \\ \frac{-a^2+a+1}{-a^2+a} & \frac{a}{-a^2+a} & \frac{-1}{-a^2+a} \\ \frac{a}{-a^2+a} & \frac{a}{-a^2+a} & \frac{-a}{-a^2+a} \end{pmatrix}$$

Podem simplificar  $\frac{a}{-a^2+a} = \frac{a}{a(-a+1)} = \frac{1}{-a+1}$ .

$$c) A = \begin{pmatrix} 4 & 3 & \lambda \\ 2 & 1 & 2 \\ \lambda & \lambda & -1 \end{pmatrix}$$

$$|A| = -4 + 6\lambda + 2\lambda^2 - \lambda^2 + 6 - 8\lambda$$

$$= \lambda^2 - 2\lambda + 2 = 0 \Rightarrow \lambda = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{2 \pm \sqrt{-4}}{2}$$

$\Rightarrow$  no real solutions.

$\Rightarrow |A| \neq 0$  sempre  $\Rightarrow A$  sempre is regular

Sempre podem achar  $A^{-1}$ .

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 1 & 2 \\ \lambda & -1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ \lambda & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ \lambda & \lambda \end{vmatrix} \\ -\begin{vmatrix} 3 & \lambda \\ \lambda & -1 \end{vmatrix} & \begin{vmatrix} 4 & \lambda \\ \lambda & -1 \end{vmatrix} & -\begin{vmatrix} 4 & 3 \\ \lambda & \lambda \end{vmatrix} \\ \begin{vmatrix} 2 & \lambda \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 4 & \lambda \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 - 2\lambda & 2 + 2\lambda & \lambda \\ 3 + \lambda^2 & -4 - \lambda^2 & -\lambda \\ 6 - \lambda & -8 + 2\lambda & -2 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-1 - 2\lambda}{\lambda^2 + 2} & \frac{2 + 2\lambda}{\lambda^2 + 2} & \frac{6 - \lambda}{\lambda^2 + 2} \\ \frac{3 + \lambda^2}{\lambda^2 + 2} & \frac{-4 - \lambda^2}{\lambda^2 + 2} & \frac{-8 + 2\lambda}{\lambda^2 + 2} \\ \frac{\lambda}{\lambda^2 + 2} & \frac{-\lambda}{\lambda^2 + 2} & \frac{-2}{\lambda^2 + 2} \end{pmatrix}$$

$$b) A = \begin{pmatrix} 2 & 1 & -a \\ 2a & 1 & -1 \\ 2 & a & 1 \end{pmatrix}$$

$$|A| = \cancel{2} - \cancel{2} - 2a^3 + 2a - \cancel{2}a + \cancel{2}a$$

$$= -2a^3 + 2a = 0$$

$$2a(-a^2 + 1) = 0$$

$$\begin{cases} \rightarrow a = 0 \\ \rightarrow -a^2 + 1 = 0 \Rightarrow a^2 = 1 \Rightarrow a = \pm 1 \end{cases}$$

$$\begin{cases} -2a^3 + 2a = \\ = -2a(a^2 - 1) \\ = -2a(a-1)(a+1) \end{cases}$$

Si  $a \neq 0$ ,  $a \neq 1$  i  $a \neq -1 \Rightarrow A$  es regular.

Calcular  $A^{-1}$  en cofactores,

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 1 & -1 \\ a & 1 \end{vmatrix} & -\begin{vmatrix} 2a & -1 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 2a & 1 \\ 2 & a \end{vmatrix} \\ -\begin{vmatrix} 1 & -a \\ a & 1 \end{vmatrix} & \begin{vmatrix} 2 & -a \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 2 & a \end{vmatrix} \\ \begin{vmatrix} 1 & -a \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & -a \\ 2a & -1 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 2a & 1 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1+a & -2a-2 & 2a^2-2 \\ -1-a^2 & 2+2a & -2a+2 \\ -1+a & 2-2a^2 & 2-2a \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1+a}{-2a(a-1)(a+1)} & \frac{-1-a^2}{-2a(a-1)(a+1)} & \frac{-1+a}{-2a(a-1)(a+1)} \\ \frac{-2a-2}{-2a(a-1)(a+1)} & \frac{2+2a}{-2a(a-1)(a+1)} & \frac{2-2a}{-2a(a-1)(a+1)} \\ \frac{-2a^2-2}{-2a(a-1)(a+1)} & \frac{-2a+2}{-2a(a+1)(a-1)} & \frac{2-2a}{-2a(a-1)(a+1)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{-2a(a-1)} & \frac{a^2+1}{2a(a-1)(a+1)} & \frac{1}{-2a(a+1)} \\ \frac{1}{a(a-1)} & \frac{1}{-a(a-1)} & \frac{1}{a} \\ \frac{1}{a} & \frac{1}{a(a-1)} & \frac{1}{a(a+1)} \end{pmatrix}$$

$$k) A = \begin{pmatrix} -1 & -1 & 2 \\ k & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$|A| = \cancel{1} + 2k + k + \cancel{1} = 3k$$

Si  $k \neq 0 \Rightarrow A$  es invertible.

En aquest cas, calculem  $A^{-1}$ .

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} k & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} k & 0 \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & +2 \\ k & 1 \end{vmatrix} & \begin{vmatrix} -1 & -1 \\ k & 0 \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -k+1 & k \\ 3 & -3 & 0 \\ -1 & 1+2k & k \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-1}{3k} & \frac{1}{k} & \frac{-1}{3k} \\ \frac{-k+1}{3k} & -\frac{1}{k} & \frac{1+2k}{3k} \\ \frac{1}{3} & 0 & \frac{1}{3} \end{pmatrix}$$

$$e) A = \begin{pmatrix} a & 1 & 2 \\ a & a & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

$$|A| = a^2 + 2 + 2a - \cancel{2a} - a - 2a$$

$$= a^2 - 3a + 2 = 0$$

$$a = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{3 \pm 1}{2} \begin{matrix} 2 \\ 1 \end{matrix}$$

$\Rightarrow$  Si  $a \neq 1$  i  $a \neq 2 \Rightarrow A$  és regular.

Calcular, en aquest cas,  $A^{-1}$ .

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} a & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} a & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} a & a \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} a & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} a & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix} & -\begin{vmatrix} a & 2 \\ a & 2 \end{vmatrix} & \begin{vmatrix} a & 1 \\ a & a \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a-2 & -a+2 & 0 \\ 1 & a-2 & -a+1 \\ 2-2a & 0 & a^2-a \end{pmatrix}$$



$$A^{-1} = \begin{pmatrix} \frac{a^2}{(a-1)(a-2)} & \frac{1}{(a-1)(a-2)} & \frac{-2(a-1)}{(a-1)(a-2)} \\ \frac{-(a-2)}{(a-1)(a-2)} & \frac{a^2}{(a-1)(a-2)} & 0 \\ 0 & \frac{-(a-1)}{(a-1)(a-2)} & \frac{a(a-1)}{(a-1)(a-2)} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{a-1} & \frac{1}{(a-1)(a-2)} & \frac{-2}{a-2} \\ \frac{-1}{a-1} & \frac{1}{a-1} & 0 \\ 0 & \frac{-1}{a-2} & \frac{a}{a-2} \end{pmatrix}$$

$$m) A = \begin{pmatrix} a & 1 & a \\ 2 & a & 2 \\ a & 1 & 1 \end{pmatrix}$$

$$\begin{aligned} |A| &= a^2 + 2a + 2a - a^3 - 2 - 2a \\ &= -a^3 + a^2 + 2a - 2 = 0 \end{aligned}$$

$$D_{\sqrt{2}}(2) = \{ \pm 1, \pm 2 \}$$

$$\begin{array}{c|cccc} & -1 & 1 & 2 & -2 \\ \hline 1 & -1 & 0 & 2 & 2 \\ & -1 & 0 & 2 & \boxed{0} \end{array}$$

$$\begin{aligned} -a^3 + a^2 + 2a - 2 \\ = (a-1)(-a^2+2) \end{aligned}$$

$$-a^2 + 2 = 0$$

$$\Rightarrow a = \pm \sqrt{2}$$

Si  $a \neq 1$  y  $a \neq \sqrt{2}$ ,  $a \neq -\sqrt{2} \Rightarrow A$  es invertible.

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} a & 2 \\ 1 & 1 \end{vmatrix} & - \begin{vmatrix} 2 & 2 \\ a & 1 \end{vmatrix} & \begin{vmatrix} 2 & a \\ a & 1 \end{vmatrix} \\ - \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} a & a \\ a & 1 \end{vmatrix} & - \begin{vmatrix} a & 1 \\ a & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & a \\ a & 2 \end{vmatrix} & - \begin{vmatrix} a & a \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} a & 1 \\ 2 & a \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a-2 & -2+2a & 2-a^2 \\ a-1 & a-a^2 & 0 \\ 2-a^2 & 0 & a^2-2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{a-2}{(a-1)(-a^2+2)} & \frac{1}{-a^2+2} & \frac{1}{a-1} \\ \frac{2}{-a^2+2} & \frac{-a}{-a^2+2} & 0 \\ \frac{1}{a-1} & 0 & \frac{-1}{a-1} \end{pmatrix}$$

$$n) A = \begin{pmatrix} a & 1 & 2 \\ 2 & a & 2 \\ a & 1 & a \end{pmatrix}$$

$$|A| = a^3 + 2a + 4 - 2a^2 - 2a - 2a \\ = a^3 - 2a^2 - 2a + 4 = 0$$

$$\text{Div}(4) = \{ \pm 1, \pm 2, \pm 4 \}$$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -2 & 4 \\ \hline 1 & & 1 & -1 & -3 \\ 1 & 1 & -1 & -3 & 1 \end{array} \quad \begin{array}{c|cccc} 1 & 1 & -2 & -2 & 4 \\ \hline -1 & & -1 & 3 & -1 \\ 1 & 1 & -3 & 1 & 3 \end{array}$$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -2 & 4 \\ \hline 2 & & 2 & 0 & -4 \\ 1 & 1 & 0 & -2 & 0 \end{array}$$

$\Rightarrow$  2 es ermel

$$a^3 - 2a^2 - 2a + 4 \\ = (a-2)(a^2-2)$$

$$= (a-2)(a-\sqrt{2})(a+\sqrt{2})$$

$\Rightarrow$  Si  $a \neq 2, a \neq \sqrt{2}, a \neq -\sqrt{2} \Rightarrow A$  es  
mijlan.

En aquest casos, calculam  $A^{-1}$ .

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} a & 2 \\ 1 & a \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ a & a \end{vmatrix} & \begin{vmatrix} 2 & a \\ a & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 2 \\ 1 & a \end{vmatrix} & \begin{vmatrix} a & 2 \\ a & a \end{vmatrix} & -\begin{vmatrix} a & 1 \\ a & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix} & -\begin{vmatrix} a & 2 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} a & 1 \\ 2 & a \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} a^2-2 & 0 & 2-a^2 \\ -(a-2) & a^2-2a & 0 \\ 2-2a & -(2a-4) & a^2-2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{a-2} & \frac{-1}{a^2-2} & \frac{2-2a}{(a-2)(a^2-2)} \\ 0 & \frac{a}{a^2-2} & \frac{-2}{a^2-2} \\ \frac{-1}{a-2} & 0 & \frac{1}{a-2} \end{pmatrix}$$

$$a) A = \begin{pmatrix} a & -1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 4a \end{pmatrix}$$

$$|A| = 4a^3 - 1 - 1 - a + 4a - a$$

$$= 4a^3 + 2a = 0$$

$$2a(2a^2 + 1) = 0$$

$$\begin{cases} \rightarrow a = 0 \\ \rightarrow 2a^2 + 1 = 0 \end{cases}$$

$$2a^2 + 1 = 0$$

$$a^2 = \pm \sqrt{-\frac{1}{2}} \text{ no té solució.}$$

Si  $a \neq 0 \Rightarrow A$  és regular.

En aquest cas, podem calcular  $A^{-1}$ .

$$\text{Adj}(A) = \begin{pmatrix} \begin{vmatrix} a & 1 \\ 1 & 4a \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 4a \end{vmatrix} & \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ 1 & 4a \end{vmatrix} & \begin{vmatrix} a & 1 \\ 1 & 4a \end{vmatrix} & -\begin{vmatrix} a & -1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ a & 1 \end{vmatrix} & -\begin{vmatrix} a & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} a & -1 \\ 1 & a \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 4a^2 - 1 & -4a + 1 & 1 - a \\ 4a + 1 & 4a^2 - 1 & -a - 1 \\ -1 - a & -a + 1 & a^2 + 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{4a^2 - 1}{4a^3 + 2a} & \frac{-4a + 1}{4a^3 + 2a} & \frac{1 - a}{4a^3 + 2a} \\ \frac{-4a + 1}{4a^3 + 2a} & \frac{4a^2 - 1}{4a^3 + 2a} & \frac{-a - 1}{4a^3 + 2a} \\ \frac{1 - a}{4a^3 + 2a} & \frac{-a - 1}{4a^3 + 2a} & \frac{a^2 + 1}{4a^3 + 2a} \end{pmatrix}$$

$$p) A = \begin{pmatrix} k & -1 & 1 \\ k & k & 1 \\ 1 & -1 & k \end{pmatrix}$$

$$\begin{aligned} |A| &= k^3 - 1 - k - k + k^2 + k \\ &= k^3 + k^2 - k - 1 = 0 \end{aligned}$$

$$\text{Div}(A) = \sum \pm 1j \quad \left| \begin{array}{cccc} 1 & 1 & -1 & -1 \\ 1 & 1 & 2 & 1 \\ \hline 1 & 2 & 1 & 0 \end{array} \right.$$

$$\Rightarrow |A| = (k-1)(k^2+2k+1)$$

$$= (k-1)(k+1)^2 = 0$$

$$\begin{cases} \rightarrow k = 1 \\ \rightarrow k = -1 \end{cases}$$

Si  $k \neq \pm 1 \Rightarrow A$  es regular.

$$Adj(A) = \begin{pmatrix} \begin{vmatrix} k & 1 \\ -1 & k \end{vmatrix} & -\begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} & \begin{vmatrix} k & k \\ 1 & -1 \end{vmatrix} \\ -\begin{vmatrix} -1 & 1 \\ -1 & k \end{vmatrix} & \begin{vmatrix} k & 1 \\ 1 & k \end{vmatrix} & -\begin{vmatrix} k & -1 \\ 1 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 1 \\ k & 1 \end{vmatrix} & -\begin{vmatrix} k & 1 \\ k & 1 \end{vmatrix} & \begin{vmatrix} k & -1 \\ k & k \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} k^2+1 & -(k^2-1) & -2k \\ k-1 & k^2-1 & k-1 \\ -(k+1) & 0 & k(k+1) \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{k^2+1}{(k-1)(k+1)^2} & \frac{1}{(k+1)^2} & \frac{-1}{(k-1)(k+1)} \\ \frac{-1}{k+1} & \frac{1}{k+1} & 0 \\ \frac{-2k}{(k-1)(k+1)^2} & \frac{1}{(k+1)^2} & \frac{k}{(k-1)(k+1)} \end{pmatrix}$$

$\sqrt{k^2-1} = (k-1)(k+1)$

98) Calculez el rango de cada una de las matrices siguientes.

$$a) A = \begin{pmatrix} 1 & 4 & -1 \\ -1 & 3 & 2 \\ 2 & 2 & 0 \end{pmatrix}$$

$$|A| = 16 + 2 + 6 - 4 = 14 \neq 0$$

Como que  $|A| \neq 0 \Rightarrow$  tiene ~~todas~~ un menor no nulo de orden 3  $\Rightarrow \text{rg } A = 3$

$$b) B = \begin{pmatrix} 1 & -2 & 0 & 3 \\ -1 & 3 & 1 & 4 \\ 2 & 1 & 5 & 1 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ -1 & 3 & 1 \\ 2 & 1 & 5 \end{vmatrix} = 15 - 4 + 10 - 1 = 20 \neq 0$$

tiene ~~todas~~ un menor no nulo de orden 3  $\Rightarrow \text{rg } B \geq 3$

$$\neq \text{rg } B \leq \min\{3, 4\} = 3$$

$$\Rightarrow \text{rg } B = 3.$$

$$c) C = \begin{pmatrix} 3 & 5 & 1 \\ 6 & 10 & -2 \\ 1 & 0 & 1 \\ 4 & 5 & 0 \end{pmatrix}$$

$$\text{Siguiendo } \Delta_1 = \begin{vmatrix} 6 & 10 & -2 \\ 1 & 0 & 1 \\ 4 & 5 & 0 \end{vmatrix} = 40 - 10 - 30 = 0 \Rightarrow$$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & 1 \\ 6 & 10 & -2 \\ 1 & 0 & 1 \end{vmatrix} = 30 - 10 - 10 - 30 \\ = -20 \neq 0$$

Compte que  $\Delta_2 \neq 0 \Rightarrow \text{rg } C = 3$

$$d) D = \begin{pmatrix} 1 & -2 & 0 & 3 \\ -1 & 3 & 1 & 4 \\ 0 & 1 & 1 & 7 \end{pmatrix}$$

Sabem que  $\text{rg } D \leq 3$

Minem si pot ser 3.

$$\text{Sigui } \Delta_1 = \begin{vmatrix} 1 & -2 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 3 - 2 - 1 = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 3 & 4 \\ 0 & 1 & 7 \end{vmatrix} = 21 - 3 - 14 - 4 \\ = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 0 & 3 \\ -1 & 1 & 4 \\ 0 & 1 & 7 \end{vmatrix} = 7 - 3 - 4 = 0$$

$$\Delta_4 = \begin{vmatrix} -2 & 0 & 3 \\ 3 & 1 & 4 \\ 1 & 1 & 7 \end{vmatrix} = -14 + 9 - 3 + 8 \\ = 0$$

Compte tots els menors d'ordre 3 són iguals a 0  
 $\Rightarrow \text{rg } D \neq 3$ .



Minim  $\Rightarrow$   $\text{rg } D = 2$ .

$$\begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$$

$$\Rightarrow \text{rg } D = 2$$

99) Estudie el rang de las matrices segun el valor del parámetro que le aparece:

$$a) A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & -2 \\ 3 & 1 & a \end{pmatrix}$$

$$|A| = 2a - 6 - a + 4 = a - 2$$

$$a - 2 = 0 \Rightarrow a = 2$$

• Si  $a \neq 2 \Rightarrow |A| \neq 0 \Rightarrow \text{rg } A = 3$

• Si  $a = 2 \Rightarrow |A| = 0 \Rightarrow \text{rg } A \neq 3$ .

Minim  $\Rightarrow$   $\text{rg } A = 2$  o mes petit:

$$\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1 \neq 0$$

$$\Rightarrow \text{rg } A = 2$$

En conclusió si  $a \neq 2 \Rightarrow \text{rg } A = 3$

si  $a = 2 \Rightarrow \text{rg } A = 2$ .

$$b) B = \begin{pmatrix} 2 & 1 & a \\ a & 3 & 4 \\ 3 & -1 & 2 \end{pmatrix}$$

$$|B| = 12 + 12 - a^2 - 9a - 2a + 8$$

$$= -a^2 - 11a + 32$$

$$-a^2 - 11a + 32 = 0$$

$$\Rightarrow a = \frac{11 \pm \sqrt{121 - 4 \cdot (-1) \cdot 32}}{2 \cdot (-1)}$$

$$= \frac{11 \pm \sqrt{249}}{-2}$$

• Si  $a \neq \frac{11 \pm \sqrt{249}}{-2} \Rightarrow |B| \neq 0 \Rightarrow \text{rg } B = 3$

• Si  $a = \frac{11 + \sqrt{249}}{-2} \Rightarrow |B| = 0 \Rightarrow \text{rg } B \neq 3$

$$\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix} = 6 + 4 = 10 \neq 0 \Rightarrow \text{rg } B = 2$$

• Si  $a = \frac{11 - \sqrt{249}}{-2} \Rightarrow |B| = 0 \Rightarrow \text{rg } B \neq 3$

$$\begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix} = 10 \neq 0 \Rightarrow \text{rg } B = 2$$

$$c) C = \begin{pmatrix} a & -1 & 1 \\ 1 & -a & 2a-1 \end{pmatrix}$$

Sabem que  $\text{rg } C \leq 2$

$$\text{Si} \Delta = \begin{vmatrix} a & -1 \\ 1 & -a \end{vmatrix} = -a^2 + 1$$

$$-a^2 + 1 = 0 \Rightarrow 1 = a^2 \Rightarrow a = \pm 1$$

• Si  $a \neq +1, a \neq -1 \Rightarrow \Delta \neq 0 \Rightarrow \text{rg } C = 2$

• Si  $a = 1 \Rightarrow \Delta = 0 \Rightarrow$  No podem assegurar que  $\text{rg } C \neq 2, j^{\circ}$

que hi ha altres menors d'ordre 2.

$$C = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \quad (1^{\circ} \text{ i } 3^{\circ} \text{ columnes})$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \quad (2^{\circ}, 3^{\circ} \text{ columnes})$$

$\Rightarrow$  tots els menors d'ordre 2 són nuls.

$\Rightarrow \text{rg } C \neq 2$

Es veu que  $|1| = 1 \Rightarrow \text{rg } C = 1$

• Si  $a = -1 \Rightarrow A = 0$

$$C = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 1 & -3 \end{pmatrix}$$

$$\Delta a = \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 3 - 1 = 2 \neq 0$$

$$\Rightarrow \text{rg } C = 2$$

En conclusi3,

• Si  $a \neq \pm 1 \Rightarrow \text{rg } C = 2$

• Si  $a = 1 \Rightarrow \text{rg } C = 1$

• Si  $a = -1 \Rightarrow \text{rg } C = 2$

d)  $D = \begin{pmatrix} t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & t \end{pmatrix}$

$$\begin{aligned} |D| &= -t^3 + 1 + 1 + t - t - t \\ &= -t^3 - t + 2 = 0 \end{aligned}$$

$$\text{Div}(2) = \{ \pm 1, \pm 2 \}$$

$$\begin{array}{r|rrrr} & -1 & 0 & -1 & 2 \\ \hline 1 & & -1 & -1 & -2 \\ \hline & -1 & -1 & -2 & 0 \end{array}$$

1 es arrel

$$-t^3 - t + 2 =$$

$$(t-1)(-t^2-t-2)$$

$$-t^2 - t - 2 = 0 \Rightarrow t = \frac{1 \pm \sqrt{1 - 4(-1)(-2)}}{2(-1)}$$

$$= \frac{1 \pm \sqrt{-7}}{-2} \quad \text{no real values}$$

• Si  $t \neq 1 \Rightarrow |\Delta| \neq 0 \Rightarrow \text{rg } \Delta = 3$

• Si  $t = 1 \Rightarrow |\Delta| = 0 \Rightarrow \text{rg } \Delta \neq 3$

$$\Delta = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1 - 1 = -2 \neq 0$$

$$\Rightarrow \text{rg } \Delta = 2.$$

$$e) E = \begin{pmatrix} t & 2 & 2 \\ 2 & t & 0 \\ 1 & t & t \end{pmatrix}$$

$$|E| = t^3 + 4t - 2t - 4t = t^3 - 2t = 0$$

$$\Rightarrow t(t^2 - 2) = 0 \Rightarrow t = 0$$

$$\Rightarrow t = \pm\sqrt{2}$$

• Si  $t \neq 0$  i  $t \neq \pm\sqrt{2} \Rightarrow |E| \neq 0 \Rightarrow \text{rg } E = 3$

$$\bullet \text{ Si } t = 0 \Rightarrow E = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4 \neq 0 \Rightarrow \text{rg } E = 2$$

$$\bullet \text{ si } t = \sqrt{2} \Rightarrow E = \begin{pmatrix} \sqrt{2} & 2 & 2 \\ 2 & \sqrt{2} & 0 \\ 1 & \sqrt{2} & \sqrt{2} \end{pmatrix}$$

$$\begin{vmatrix} \sqrt{2} & 0 \\ \sqrt{2} & \sqrt{2} \end{vmatrix} = 2 \neq 0 \Rightarrow \text{rg } E = 2$$

$$\bullet \text{ si } t = -\sqrt{2} \Rightarrow E = \begin{pmatrix} -\sqrt{2} & 2 & 2 \\ 2 & -\sqrt{2} & 0 \\ 1 & -\sqrt{2} & -\sqrt{2} \end{pmatrix}$$

$$\begin{vmatrix} -\sqrt{2} & 0 \\ -\sqrt{2} & -\sqrt{2} \end{vmatrix} = 2 \neq 0 \Rightarrow \text{rg } E = 2$$

En conclusio, si  $t \neq 0$ ,  $t \neq \pm\sqrt{2} \Rightarrow \text{rg } E = 3$   
 altrament,  $\text{rg } E = 2$ .

$$2) F = \begin{pmatrix} t+3 & 4 & 0 \\ 0 & t-1 & 1 \\ -4 & -4 & t-1 \end{pmatrix}$$

$$|F| = (t-1)(t+3) - 16 + 4(t+3)$$

$$= t^2 + 3t - t - 3 - 16 + 4t + 12$$

$$= t^2 + 6t - 7 = 0 \Rightarrow t = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot (-7)}}{2 \cdot 1}$$

$$= \frac{-6 \pm 8}{2} \begin{matrix} 1 \\ -7 \end{matrix}$$

• Si  $t \neq 1$  e  $t \neq -7 \Rightarrow \text{rg } F = 3$  perchè  $|F| \neq 0$

• Si  $t = 1 \Rightarrow |F| = 0 \Rightarrow \text{rg } F \neq 3$ .

$$F = \begin{pmatrix} 4 & 4 & 0 \\ 0 & 0 & 1 \\ -4 & -4 & 0 \end{pmatrix}$$

$$\begin{vmatrix} 0 & 1 \\ -4 & 0 \end{vmatrix} = 0 + 4 = 4 \neq 0 \Rightarrow \text{rg } F = 2$$

• Si  $t = -7 \Rightarrow |F| = 0 \Rightarrow \text{rg } F \neq 3$

$$F = \begin{pmatrix} -4 & 4 & 0 \\ 0 & -8 & 1 \\ -4 & -4 & -8 \end{pmatrix}$$

$$\begin{vmatrix} -8 & 1 \\ -4 & -8 \end{vmatrix} = 64 + 4 = 68 \neq 0 \Rightarrow \text{rg } F = 2$$

$$f) G = \begin{pmatrix} t & 1 & 1 & 2 \\ 2 & t & t^2 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

Saperi che  $\text{rg } G \leq 3$

$$\Delta = \begin{vmatrix} t & 1 & 2 \\ 2 & t & 1 \\ 2 & 1 & 2 \end{vmatrix} = 2t^2 + 2 + 2 - 4t - 4 - t$$
$$= 2t^2 - 5t = 0$$

$$\Rightarrow t(2t - 5) = 0$$

$$\Rightarrow t = 0$$

$$\hookrightarrow t = \frac{5}{2}$$

• Si  $t \neq 0$  :  $t \neq \frac{5}{2} \Rightarrow \Delta \neq 0 \Rightarrow \text{rg } G = 3$

• Si  $t = 0 \Rightarrow \Delta = 0$

$$G = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 0 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix} = 2 + 4 - 4 - 1 \neq 0$$

$$\Rightarrow \text{rg } G = 3$$

• Si  $t = \frac{5}{2}$

$$\Rightarrow G = \begin{pmatrix} \frac{5}{2} & 1 & 1 & 2 \\ 2 & \frac{5}{2} & \frac{25}{4} & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

$$\Delta_2 = \begin{vmatrix} \frac{5}{2} & 1 & 1 \\ 2 & \frac{5}{2} & \frac{25}{4} \\ 2 & 1 & 1 \end{vmatrix} = \frac{25}{4} + \frac{25}{2} + 2 - 5 - 2 - \frac{125}{8} = -\frac{15}{8} \neq 0$$

$$\Rightarrow \text{rg } G = 3$$



100) Étudiez le rang de la matrice suivant en fonction de  $a, b, c$ :

$$A = \begin{pmatrix} 5 & 5 & 5 \\ a & b & c \\ b+c & a+c & a+b \end{pmatrix}$$

$$\begin{aligned} |A| &= 5b(a+b) + 5c(b+c) + 5a(a+c) \\ &\quad - 5b(b+c) - 5a(a+b) - 5c(a+c) \\ &= \cancel{5ab} + \cancel{5b^2} + 5bc + 5c^2 + \cancel{5a^2} + 5ac \\ &\quad - \cancel{5b^2} - \cancel{5bc} - \cancel{5a^2} - \cancel{5ab} - \cancel{5ac} - \cancel{5c^2} \\ &= 0 \end{aligned}$$

$$\Rightarrow \text{rg } A \neq 3$$

$$\text{Signi } \Delta = \begin{vmatrix} 5 & 5 \\ a & b \end{vmatrix} = 5b - 5a.$$

$$5b - 5a = 0 \Rightarrow a = b.$$

$$\bullet \text{ Si } a \neq b \Rightarrow \Delta \neq 0 \Rightarrow \text{rg } A = 2$$

$$\text{si } a = b \Rightarrow \Delta = 0$$

$$A = \begin{pmatrix} 5 & 5 & 5 \\ a & a & c \\ a+c & a+c & 2a \end{pmatrix}$$

$$\text{Signi } \Delta_2 = \begin{vmatrix} 5 & 5 \\ a & c \end{vmatrix} = 5c - 5a = 0$$

$$\Rightarrow c = a.$$

↳ si  $a \neq c \Rightarrow \Delta_2 \neq 0 \Rightarrow \text{rg } A = 2$

↳ si  $a = c \Rightarrow \Delta_2 = 0$

$$A = \begin{pmatrix} 5 & 5 & 5 \\ a & a & a \\ 2a & 2a & 2a \end{pmatrix}$$

Clarament, tenim que tots els menors d'ordre 2 són 0, ja que tenen dues columnes iguals.

$\Rightarrow \text{rg } A \neq 2$

$$|5| = 5 \neq 0 \Rightarrow \text{rg } A = 1.$$

Per tant, en resum, si  $a = b = c \Rightarrow \text{rg } A = 1$   
altrament,  $\text{rg } A = 2$ .