

## DETERMINANTS

66) Calculate the determinants given.

$$\begin{aligned} \text{a)} \quad \begin{vmatrix} -3 & -1 \\ 2 & -1 \end{vmatrix} &= -3 \cdot (-1) - 2 \cdot (-1) \\ &= +3 + 2 = \boxed{5} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad \begin{vmatrix} 10 & -5 & 3 \\ -3 & 2 & 0 \\ 4 & -3 & -2 \end{vmatrix} &= 10 \cdot 2 \cdot (-2) + (-5) \cdot 0 \cdot 4 + 3 \cdot (-3) \cdot (-3) \\ &\quad - 3 \cdot 2 \cdot 4 - (-5) \cdot (-3) \cdot (-2) - 10 \cdot 0 \cdot (-3) \\ &= -40 + 0 + 27 - 24 + 30 + 0 \\ &= \boxed{-7} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad \begin{vmatrix} 2 & 1 & -2 \\ -3 & 0 & 2 \\ 4 & -1 & 3 \end{vmatrix} &= 2 \cdot 0 \cdot 3 + 1 \cdot 2 \cdot 4 + (-2) \cdot (-3) \cdot (-1) \\ &\quad - (-2) \cdot 0 \cdot 4 - 1 \cdot (-3) \cdot 3 - 2 \cdot 2 \cdot (-1) \\ &= 0 + 8 - 6 + 0 + 9 + 4 \\ &= \boxed{15} \end{aligned}$$

$$\text{d)} \quad \begin{vmatrix} 7 & -2 \\ 1 & 4 \end{vmatrix} = 7 \cdot 4 - 1 \cdot (-2) = 7 \cdot 4 + 2 = 28 + 2 = \boxed{30}$$

$$\text{e)} \quad \begin{vmatrix} 4 & 3 & 2 \\ 1 & 2 & 4 \\ 6 & 7 & 2 \end{vmatrix} = 16 + 72 + 14 - 24 - 6 - 112 = \boxed{-40}$$

$$d) \begin{vmatrix} -2 & 3 & 7 \\ 2 & 4 & 6 \\ 2 & 5 & 3 \end{vmatrix} = -24 + 36 + 70 - 56 - 18 + 60 \\ = \boxed{68}$$

$$e) \begin{vmatrix} 3-x & 4x^2 \\ 6 & 7+2x \end{vmatrix} = (3-x) \cdot (7+2x) - 6 \cdot 4x^2 \\ = 21 + 6x - 7x - 2x^2 - 24x^2 \\ = \boxed{-26x^2 - 1x + 21}$$

$$h) \begin{vmatrix} x+1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & x+1 \end{vmatrix} = (x+1) \cdot (x+1) \cdot (x+1) + 1 + 1 \\ - 1 \cdot (x+1) \cdot 1 - 1 \cdot 1 \cdot (x+1) \\ - (x+1) \cdot 1 \cdot 1$$

$$= x^3 + 3x^2 + 3x + 1 + 1 + 1 - (x+1) - (x+1) - (x+1)$$

$$= x^3 + 3x^2 + 3x + 3 - 3(x+1)$$

$$= x^3 + 3x^2 + \cancel{3x} + \cancel{3} - \cancel{3x} - \cancel{3} = \boxed{x^3 + 3x^2}$$

$$\lceil (x+1) \cdot (x+1) \cdot (x+1) = (x^2 + x + x + 1) \cdot (x+1)$$

$$= (x^2 + 2x + 1) \cdot (x+1)$$

$$= x^3 + 2x^2 + x + x^2 + 2x + 1$$

$$= x^3 + 3x^2 + 3x + 1 \rceil$$